Cooperator-driven and defector-driven punishments: How do they influence cooperation?

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Economic studies have shown that there are two types of regulation schemes which can be considered as a vital part of today's global economy: self-regulation enforced by self-regulation organizations to govern industry practices and government regulation which is considered as another scheme to sustain corporate adherence. An outstanding problem of particular interest is to understand quantitatively the role of these regulation schemes in evolutionary dynamics. Typically, punishment usually occurs for enforcement of regulations. Taking into account both types of punishments to influence the regulations, we develop a game model where six evolutionary situations with corresponding combinations of strategies are considered. Furthermore, a semianalytical method is developed to allow us to give accurate estimations of the boundaries between the phases of full defection and nondefection. We find that, associated with the evolutionary dynamics, for an infinite well-mixed population, the mix of both punishments performs better than one punishment alone in promoting public cooperation, but for a networked population the cooperator-driven punishment turns out to be a better choice. We also reveal the monotonic facilitating effects of the synergy effect, punishment fine, and feedback sensitivity on the public cooperation for an infinite well-mixed population. Conversely, for a networked population an optimal intermediate range of feedback sensitivity is needed to best promote punishers' populations. Overall, a networked structure is overall more favorable for punishers and further for public cooperation, because of both network reciprocity and mutualism between punishers and cooperators who do not punish defectors. We provide physical understandings of the observed phenomena, through a detailed statistical analysis of frequencies of different strategies and spatial pattern formations in different evolution situations. These results provide valuable insights into how to select and enforce suitable regulation measures to let public cooperation remain prevalent, which has potential implications not only for self-regulation, but also for other topics in economics and social science.

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I. INTRODUCTION

In the field of evolutionary game theory, the conventional social dilemma, i.e., the first-order social dilemma, means that the well-being of the population depends only on the level of cooperation while defection is the best choice for an individual. Besides the mechanisms to sustain or promote cooperative behaviors such as kin selection [1], reputation [2], group selection, and reciprocity [3,4], punishment has also been widely approved as an available rule to alleviate this public good problem [5,6]. Many related studies have been performed to focus on how punishment rules govern the evolution of the game systems [5,7–9]. At the same time, these studies have affirmed that punishment is a useful tool to repel defection behaviors and to facilitate cooperation of the population, through both empirical experiments and theoretical analysis. However, second-order free-riding (i.e., the second-order dilemma) arising from the fact that punishers have to bear extra substantial punishment cost is a nonignorable impediment to the evolutionary stability of punishment, since this would weaken punishers' persistent monitoring ability and sanctions on wrongdoers [10–12]. Aiming to address this issue, some researchers have sought more effective specific strategies or mechanisms [13–18].

An issue of growing interest in the research of game theory is that humans prefer pool punishment over peer punishment for maintaining the commons [19]. Unlike peer punishment, in which the punishment act is carried out by peers, measures of pool punishment are usually outsourced and carried out by a paid organization which collects punishment costs (i.e., taxes) from the cooperators who are willing to eliminate the defectors from the population [11,20]. It is convinced that these cooperators can be regarded as punishers to some extent, who can commonly share the cost of pool punishment. Within this game framework, in some cases consideration of punishment strategies can solve the second-order free-rider problem in the presence of a segregation of behavioral strategies [21] or

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if the punishment fine is large enough [22]. Also, it has been found that prosocial punishers can outperform a combination of positive and negative reciprocity, which is able to invade defectors [23]. In particular, recent studies highlight that adaptive punishment is good at facilitating public cooperation or is even double edged, meaning that the punishers condition their sanctioning activities against antisocial behaviors on one threshold relating to the success or abundance of cooperators and defectors within the groups [24–26]. In reality, pool punishment is widely exploited by many authorities to mitigate the free riders' destructive potential, regardless of whether the punishment is direct, indirect, first order, or second order [11,27–29]. The cost of pool punishment is commonly shared, which thus could reduce both financial burdens and the risk of revenge as much as possible [30]. Implementation of institutions of pool punishment is also easily established to ensure fairness so that defectors are identified expediently and punished. As operational institutions of pool punishment, third organizations such as modern courts, the police system, and regulation organizations are developed to carry out punitive measures so as to alleviate the problems of second-order free riders, antisocial punishment [9,31], and retaliation [30]. Therefore, pool punishment has gained a great deal of attention as an important symbol of modern civilized society. Moreover, as important background to motivate our present study, there is one realistic case which must be stated with respect to punishment measures whose execution is strongly dependent on the abundance of free-riding behavior in the systems.

In commerce, self-regulation is an important mechanism for governing industry practices, possessing many benefits over government regulation for consumers, businesses, the government, and the economy as a whole [32]. The incentive for the private sector to undertake self-regulatory actions, i.e., to develop and comply with standards, is that they are incentivized by customer expectations and threatened by possible government regulation and critical public opinion, which actually issues statements of concern for public welfare [33]. Further, self-regulation is the process whereby an organization or company is impelled by self-regulation organization (SRO) to monitor its own adherence to legal, ethical, or safety standards, rather than a third party and independent agency such as a governmental entity monitor to enforce those standards [34]. Self-regulation is a win-win for businesses, consumers, and government. Businesses benefit from not only regulations that are predictable and reasonable (as opposed to command and control rules that are often burdensome and expensive to comply with), but also more efficient enforcement approaches which allow them to better manage their scarce resources [33,35] and to increase competitiveness by improving the quality of products and services as a first-mover advantage [36]. At the same time, customer expectations are satisfied because SROs enforce rules and standards set by themselves to protect consumers, which additionally upholds rights for employees and improve public trust [32,37]. Self-implemented standards can also span jurisdictions [38]. Studies have shown that self-policing across locations makes industry-developed standards more predictable and consistent and therefore less costly than government regulations [32,39].

On the other hand, costs of self-regulation activities which are imposed on firms cause them to shift resources away from other activities to achieve compliance. These costs are often justified as a means of improving social welfare; however, they are also a negative factor giving rise to a free-rider problem which would cause incredible harm to people, government, and businesses [40]. In detail, in order to be effective, a SRO may set rules for an industry including firms that do not participate in the SRO. These outside firms enjoy the benefits of the regulatory regime without paying any of the costs, as well as those bad actors who also stay outside the system so that they can avoid the rules of the SRO. Such a system is actually unfair to dues-paying businesses, which makes self-regulation an inadequate choice for certain industries. This limitation of SROs cannot be ignored. Additionally, self-regulation ineffectively enforces its rules as a punishment tool for governing the private sector when the problem that massive firms violates rights seems to be widespread, from India's mining sector [41] to Cambodia's garment industry [42] to the debt buying industry in the U.S. [43,44]. In such cases when a high proportion of entities are found to be unlawful, even the threat of powerful measures of government regulation can lead to more effective and stronger enforcement by the SRO or public-expected results through direct government involvement. This suggests that government oversight or enforcement is indispensable or even the final guarantee for public welfare, regardless of the fact that by its nature it creates barriers to innovation or competitive entry because of its established norms that only capture current market participants and activities.

Meaningful regulation schemes are usually driven by a complex mix of internal, external, and reputational motivations [45]. In particular, the nature of intrinsic organizational motivation is central to the definition of the both regulation schemes [32,45]. The above statement of the two regulation schemes initially shows that the number or proportion of disciplined members or bad actors or wrongdoing in the SROs can be considered as crucial intrinsic motivation to drive meaningful implementations of the two regulations. This is also supported by previous empirical studies involving self-regulation [40,43,44,46] that show that the abundance of disciplined members or bad actors in the SROs is essential to influence the action modes of the two regulation measures. Note that not only punitive sanctions but also other tools such as regulatory threats and surveillance can be effective means of regulation enforcement, while measures of the two regulations are used as punitive actions in our study for the sake of exploration. This assumption is rational since punitive enforcement, or at least the possibility of it, turns out to be essential to the ultimate success of schemes that incorporate either self-regulation or government regulation [32]. Therefore, for simplicity, in the present paper we can correspondingly define self-regulation and government regulation as cooperator-driven punishment (CP) and defector-driven punishment (DP), within the framework of game theory, by virtue of one of their intrinsic motivations: the abundance of bad actors or disciplined members in the SROs. In more detail, implementation of CP and DP is significantly driven by the abundance or proportion of disciplined and undisciplined members in the SROs (we will specify how the intrinsic motivations quantitatively govern the implementation probability of the punishments in Sec. II), respectively, which is also the definition of the two punishment measures. Meanwhile, it must be stressed that enforcement tools of the two regulations, especially self-regulation, are diverse in reality. One example of self-regulation is Financial Industry Regulatory Authority (FINRA), which is subject to U.S. Securities and Exchange Commission oversight and which imposes penalties on bad brokers [47]. In addition, the other enforcement actions of self-regulation, such as being excluded from the association and/or making public the accusations, are non-negligible [48]. In accordance with the proposed definitions of the two different regulations, the regulation issue discussed above can also be well mapped to a pool punishment in which the two different punitive measures ought to be captured.

In reality, SROs operate essentially to protect the interests of individual firms or the industry as a whole, while governments are more concerned with protecting social welfare because they face the pressures from the public all the time [49]. That is to say, unlike government regulation, industry-created standards run the risk of advancing commercial interest over public interest. For the SROs, the only way is to move faster (i.e., make higher-quality standards or stricter regulations than governments do) than the government so that they can expediently avoid greater benefit losses caused by government regulation or public criticism [50]. That is why SROs are more willing to halt the irregularities at an early stage (or there are few illegal firms to be identified in SROs). However, when a high proportion of business actors of one association grow too comfortable accepting and helping to entrench a particular kind of lawlessness [41,43,44], the considerable cost of regulations may make the interests of a particular industry and society to not align; the SROs or industry associations will not collaborate to make punishment available, but rather will collude to protect vested interests instead of public interests in the absence of any external pressure from government stakeholders [51]. Such activities can thus reduce social welfare. Many of these concerns are finally allayed by independent nonprofit public-interest organizations such as government oversights and audits which could strongly monitor and enforce rules [51]. This is the essential mechanism by which cooperator-driven punishers (defector-driven punishers) make a decision to exert punishment more when the proportion of cooperators (defectors) is higher in SROs. Relating to our model, this reality suggests that our theoretical hypothesis of the function modes of the two different pool punishment measures is rational and realistic to some extent. It is thus more convincing that government organizations can be theoretically represented by defector-driven punishers while SROs can be perfectly mapped to cooperator-driven punishers.

A global apocalypse, the 2007–2009 financial crisis [68], may provide some key hints on how the two punishment measures intervened along with an increase in bad debts brought about by more bad actors and what performances they had. Figure 1 provides further empirical evidence by illustrating five key statistical characteristic quantities (see the caption of Fig. 1 for more details of these quantities): the number of disciplinary actions against firms and their employees which are brought by FINRA, the number of fines which represent sanctions exerted by FINRA for rule violations, the total amount of fines levied from individual brokers and firms, the number of member firms (i.e., FINRA-registered firms), and the regulatory activeness of the U.S. government or Federal Reserve. In the early stages before 2008, SROs such as National Association of Securities Dealers, Inc., New York Stock Exchange, or even their combination FINRA still regulated their members through execution of their enforcement programs such as great sanctions (i.e., financial penalties) [54,69]. Correspondingly, it can be observed in Fig. 1 that self-regulation activities remained flat or even at high levels outside the shaded areas [Figs. 1(a)–1(c)], along with a decrease in government deregulation in finance [see Fig. 1(e)]. However, with persistent pressure from the loan market, more and more financial companies reached a tacit understanding so as to protect and get vested interests through creating various financial innovations which greatly change the leverage. In such case, many of the companies rabidly opposed any move to make those standards mandatory or to enforce relevant legal standards more vigorously. As a result, self-regulation presented less efficiency, further leading to the crisis which finally happened along with the bankruptcy of Lehman Brothers and the acquisition of Merrill Lynch [70]. At that moment, the public became rather angry, which made the government begin to police the financial market by means of highly punitive measures against some undisciplined firms, with the assistance of FINRA [54,68]. Figures 1(a)–1(c) thus show that a valley in self-regulation interventions from FINRA occurred during the 2007–2009 financial crisis indicted by the shaded areas; however, denser diamond markers observed in the shaded area in Fig. 1(e) mean that more frequent involvement of U.S. government or Federal Reserve was reported. The regulatory measures by government may be various and not limited to punishments like more stringent acts (like Dodd-Frank Wall Street Reform and Consumer Protection Act) and judicial investigations, most of which were actually stimulated by this crisis. Nevertheless, more frequent involvement of these measures definitely revealed greater willingness of the government to regulate financial firms and their representatives at the heart of the crisis [see Fig. 1(e)]. Moreover, we can observe that considerable government policies were still released after the crisis because policy making is rather time consuming and thus always delayed. Taken together, this disastrous process clearly shows a situation in which implementation of self-regulation or cooperator-driven punishment is promoted by abundant disciplined members who are willing to share the risk and costs of execution, while government (i.e., defector-driven punisher) is more willing to make a strong intervention through defector-driven punishments when the industry is widely eroded by a large number of bad actors. In any case, the number of FINRA-registered firms gradually decreases with time, which further proves the above conclusion. This classical example further supports the statement that the number of undisciplined (or disciplined) members is the key factor that determines the intervention level of the two different punishments.

It is believed that this important function mode of pool punishment can be found in many other realistic social systems. The key fact that motivated our present work is that, with respect to prosocial pool punishment, the two punishment measures against free riding may have definitely different performances due to different evolution situations corresponding to different combinations of strategies and different

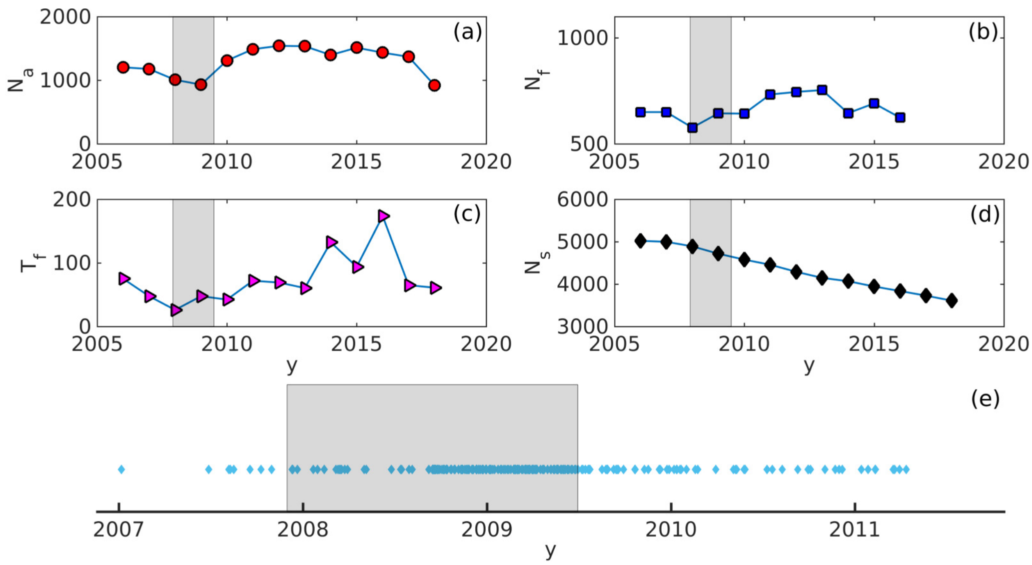

FIG. 1. Empirical evidence for the regulatory activities by either FINRA [52–65] or the U.S. government or Federal Reserve [66], which are consistent with the rules made by our model. More in detail, five key statistical characteristic quantities are shown. (a) N_a denotes the number of disciplinary actions against firms and their employees, which are brought by FINRA, the SRO for brokerage firms doing business with the public in the U.S. The disciplinary actions may result in sanctions including censures, fines, suspensions, and, in egregious cases, expulsions or bars from the industry [62]. (b) N_f denotes the number of fines which represent sanctions exerted by FINRA for rule violations. It should be noted that the number of FINRA fines for 2017 and 2018 is still either being counted or even unavailable. (c) T_f is the total number of fines levied by FINRA on individual brokers and firms. (d) Size of the whole population of registered members (i.e., FINRA-registered firms instead of disciplined members or cooperators) which gradually decreases with time. (e) Regulatory activity of the U.S. government or Federal Reserve, in which each diamond represents a press release about regulatory policy or an act made by public institutions of the U.S. government or Federal Reserve. Obviously, the time windows during which denser diamond markers can be observed reveal more frequent involvement of government regulations. In all panels, the shaded areas represent the duration of the 2007–2009 financial crisis (December 2007 to June 2009) [67]. We still cannot know accurately the frequency or number of cooperators (i.e., disciplined members) or defectors (i.e., undisciplined members) during this financial crisis because not all bad actions can be successfully identified and the cost of investigations themselves is huge. However, it must be stressed that the frequency of disciplinary actions against firms which are brought by the government or FINRA is positively related to the quantity of undisciplined actions in the course of financial crisis or at other times, respectively. Furthermore, more undisciplined members were "identified" as forms of bankruptcy, a higher nonperforming loan ratio, or being judicially investigated during the crisis than at other times [65,66,68]. Evidently, there are more identified undisciplined members in the course of the financial crisis, which thus correspondingly suggests a lower proportion or fewer cooperators at this time because of fewer registered members than before [in (d)].

population structures. For a model of evolution dynamics to capture the real behaviors as accurately as possible, the distinct probability to influence the establishment or enforcement degree of the two different punishment measures must be taken into account in the framework of pool punishment.

In addition, a widely applied game framework, the public good game (PGG) provides a good theoretical framework that concerns public welfare in the presence of pool punishment. Grouplike structure in the PGG is in favor of the function of pool punishment, even though the overlap between different game groups depends on the network structures. Another reason for PGG model being the first choice as the present game framework is that its rules are very close to the operating modes of many modern money-seeking organizations such as banks, profit funds, or listed companies: attracting capital and then sharing investment gains together.

In the current literature, there is a lack of work on evolutionary dynamics which takes into account issues of self-regulation and government regulation. In general, both types of regulations exist and the question is how important they are in governing the evolutionary dynamics in different situations. In this paper we propose a general evolutionary game model to capture the two distinct punishment measures, defector-driven and cooperator-driven punishment, in the framework of the PGG. The evolution dynamics in six different evolutionary situations on well-mixed population and networked population is treated in detail by mean-field theory and extensive agent-based simulations, respectively. Moreover, our semianalytical method is able to yield accurate estimations of the boundaries between the phases of full defection and nondefection. We provide several physical understandings of the basic evolutionary dynamics for different situations in the two populations, through a detailed statistical analysis, uncovering the favorable conditions under which cooperation prevalence with abundant punishers can arise.

Three striking phenomena are revealed in this study. One is that a networked structure is overall more favorable for punishers and further for cooperation because of network reciprocity and mutualism between punishers and cooperators who do not punish defectors. The second phenomenon is that, for networked populations, cooperator-driven punishment is a more efficient measure to confer nondefectors evolutionary advantages; however, in infinite well-mixed populations the mix of the two punishments is a better choice to achieve a desirable evolutionary outcome. Finally, of particular interest is that, for a networked population, an optimal intermediate range of feedback sensitivity for the prevalence of punishers is identified. We give a clear physical picture to help us understand how this phenomenon happens, through a detailed statistical analysis of spatial pattern formations in different evolutionary situations.

The paper is organized as follows. We first give a detailed description of our model in Sec. II. In Sec. III we first implement our model for six different evolutionary situations in infinite well-mixed population through theoretical approach and then extend our study to networked populations by means of agent-based simulations. Finally, we present a discussion and conclusions with an outlook in Sec. IV.

II. MODEL

In our model, policing involvement or regulations are directly assumed to be punitive measures so that one can design a feasible model for analytical and numerical studies of the effectiveness of two different punishments in different evolutionary situations. In reality, regulations may create costs as well as benefits from the increasing levels of disciplined behaviors, so one should consider the cost of punishment in the model in addition to the punishment fine to quantify the punitive effects in terms of high-order benefits (i.e., the emergence and persistence of cooperators or prosocial punishers). In the matter of regulation of industry, the key players in the promotion of public interests have always been businesses, SROs, government, and consumer advocates, which should be considered as basic ingredients or strategies in our present model. Accordingly, there are four strategies within the framework of the PGG, traditional cooperation (TC), i.e., nonpunishing cooperation, defection (D), cooperator-driven punishment, and defector-driven punishment, and correspondingly four types of individuals, traditional cooperators (nonpunishing cooperators), defectors, cooperator-driven punishers, and defector-driven punishers. Of particular note here is that the execution probabilities of CP and DP are mainly determined by the fractions of cooperators and defectors within the game group, respectively, which is also the concrete definition of the two punishment measures in our model. As another punishment measure, traditional punishment (TP) is usually implemented when there is at least one defector in the group and thus is widely adopted. However, we have checked that this punishment strategy remains rather vulnerable and negligible, especially when CP or DP is present. It reveals that CP and DP are effective ways to regulate defectors in the institution rather than TP, at least within the framework of the present model. Hence this punishment is not considered in our present model. For simplicity, in our model the punishment mechanism is only stated on prosocial punishment, i.e., punishers adopt a cooperation strategy before punishing defectors, which means that no other mechanisms such as antisocial punishment [71–73] or selfish punishment [17] are captured.

According to our game rule, in the population each player i selects $G_i - 1$ individuals from the population to form a game group in which each group member can simultaneously play the PGG with other group members, by holding the same strategy. In detail, in the game group each cooperator makes a contribution of 1 to the public good, while defectors contribute nothing. Subsequently, the sum of all the contributions in the group is multiplied by the synergy factor $1 < r < G_i$, which quantifies synergistic effects of cooperation. Then the resulting amount is equally shared among all members in the group. After the intervention of punishments, there are two different cases to be considered for a member i adopting cooperation strategy: (1) The payoff of i will be $\Pi_{\mathrm{P}}^{g} = r(N_{\mathrm{TC}} + N_{\mathrm{CP}} + N_{\mathrm{DP}})/G_i - 1 - N_{\mathrm{D}}\alpha/n_{\mathrm{P}}$ if the punisher carries out punishment upon defectors in the group at a probability g, as either a cooperator-driven punisher or a defector-driven punisher; (2) otherwise the payoff of i is $\Pi_{\mathrm{P}}^{g} = \Pi_{\mathrm{TC}}^{g} = r(N_{\mathrm{TC}} + N_{\mathrm{CP}} + N_{\mathrm{DP}})/G_i - 1$, which is also the payoff of traditional cooperators in the group. Herein N_{TC}, N_{D}, N_{P}, N_{CP}, and N_{DP} are, respectively, the number of traditional cooperators, defectors, punishers, cooperator-driven punishers, and defector-driven punishers in the group. Therefore, $N_{\mathrm{P}} = N_{\mathrm{CP}} + N_{\mathrm{DP}}$. Meanwhile it must be stressed that n_{P} is the number of punishments exerted by punishers rather than the total number of the two types of punishers, while α is the punishment fine that each defector in the group incurs in the presence of punishment. In the case that i is a defector, $\Pi_{\mathrm{D}}^{g} = r(N_{\mathrm{TC}} + N_{\mathrm{CP}} + N_{\mathrm{DP}})/G_i$ if $n_{\mathrm{P}} = 0$; otherwise $\Pi_{\mathrm{D}}^{g} = r(N_{\mathrm{TC}} + N_{\mathrm{CP}} + N_{\mathrm{DP}})/G_i - \alpha$. Importantly, the values of α are kept the same for cooperator-driven and defector-driven punishment so as to not give either a default evolutionary advantage or a disadvantage.

More precisely, according to the definition of DP and CP, the probability (i.e., g) that the two types of punishments are implemented by corresponding punishers is dominated by the fractions of different strategies in the group, specifically as

$$g_{\mathrm{DP}} = A\frac{N_{\mathrm{D}}}{G}, \quad g_{\mathrm{CP}} = A\frac{N_{\mathrm{C}}}{G}, \tag{1}$$

where g_{CP} and g_{DP} indicate the probability at which cooperator-driven and defector-driven punishers carry out punishment, respectively. Here $N_{\mathrm{C}} = N_{\mathrm{TC}} + N_{\mathrm{CP}} + N_{\mathrm{DP}}$ is the total number of nondefectors (including traditional cooperators, cooperator-driven cooperators, and defector-driven cooperators) in the group. The parameter $A \in [0,\ 1]$ quantifies the punishers' feedback sensitivity. In more detail, a larger value indicates more sensitive punishers and larger difference between the two types of punishers in terms of their behavior modes, and thus more punishments exerted with respect to the same fractions of defectors (nondefectors) in the group.

Furthermore, Eq. (1) reveals that the two types of punishers have two opposite feedback modes. More precisely, defector-driven punishers prefer to implement punishment to bring the population back from the brink of collapse caused by abundant defectors, regardless of the vast amount of punishment costs which could greatly reduce their payoffs. In contrast, cooperator-driven punishers are more likely to take actions insofar as a good number of nondefectors can share the costs induced by the punishment, with the purpose of reserving their payoffs first. To some extent DP and CP construct a different kind of social dilemma with traditional cooperators other than the traditional dilemma consisting of traditional cooperators and defectors. Being prudent, traditional cooperators play the role of second-order free riders because they consequently preserve higher payoffs than those punishers while doing nothing to fight against wrongdoers.

In what follows, we will explore the evolutionary dynamics through both mean-field theory concerning the infinite well-mixed situation and agent-based simulations in structured populations under various parameter conditions. We have checked that localized interactions on the square lattice with $\langle k \rangle = 4$ can lead to obvious inconsistencies between analytical predictions and simulations, by means of both great evolutionary advantages conferred to the nondefectors and considerable critical slowing down of the system. Moreover, eliminating critical slowing down of the system with fewer connections is difficult and rather time consuming. Therefore, the structured population in our study is instead curved with a regular lattice with mean degree $\langle k \rangle = 6$ and with periodic conditions while maintaining the findings from our model. On a network of size N, an overlapping game group contains all the nearest neighbors of the focal individual in addition to itself, where each individual simultaneously plays the game. At the same time, each player i holds a PGG played by $G_i = k_i + 1$ group members (together with all i's neighbors), in addition to participating in k_i games initiated by i's neighbors, where k_i is the number of the focal individual's neighbors (i.e., the degree). Therefore, each individual i simultaneously plays $k_i + 1$ PGGs by holding the same strategy.

Furthermore, in the population structure Monte Carlo simulation is employed to update the strategies of players. Random sequential updating is implemented as the updating scheme to control the evolution. Initially, each player fixed on the network is randomly and independently designated as a traditional cooperator, a defector, a cooperator-driven punisher, or a defector-driven punisher. Each time step consists of N following steps such that every player can update its strategy once on average. A randomly selected player i accumulates its overall payoff Π_{s_i} by playing the PGG in all the G_i groups as a member, where Π_{s_i} is thus the sum of all the payoffs $\Pi_{s_i}^g$ acquired from each individual group. The randomly chosen nearest neighbor j also obtains its overall payoff Π_j in the same way. Then i simulates the strategy of j with probability given by the Fermi study function $W_{j \leftarrow i} = 1/\{1 + \exp[(\Pi_i - \Pi_j)/\kappa]\}$. The function implies that players possessing higher payoffs are advantaged, while the adoption of a strategy of a player performing worse is still possible. Here κ influences the noise of the uncertainty in the adoption. Without loss of generality, we set $\kappa = 0.1$ throughout this paper. The simulations are performed until the system reaches a stationary state, i.e., the populations of different strategies become time independent or defectors go extinct.

The final densities of different strategies are averaged over 200 independent realizations to ensure low variability. The size of the network is $N = 200 \times 200$.

III. RESULTS

Before presenting the main results, we should state that the focus of the present study is on evolutionary dynamics under various parameter conditions for six different combinations of strategies: D + CP, D + DP, TC + D + CP, TC + D + DP, D + CP + DP, and TC + D + CP + DP. Both analytical treatment based on mean-field theory and agent-based simulations for networked populations are employed to enable a full exploration. Correspondingly, for comparison, we present in Sec. III A analytical predictions from infinite well-mixed populations and in Sec. III B networked population embedded on a regular lattice network.

A. Infinite well-mixed populations

The calculations reported in Appendix A give the following results for different evolutionary situations, which theoretically provide a complete picture of the model behavior. Accordingly, Fig. 2 gives a complete picture of the effects of synergistic effects of cooperation, punishment fine, and individual sensitivity on the evolution direction of the system. Overall, larger r, α, and A can confer on the punishers more evolutionary advantages, leading to the decrease of position values p_{CP} or p_{DP}. In particular, this result reveals that more sensitive punishers are able to protect the population from being corroded by defectors through more available punishment. At the same time, the middle and right panels of Figs. 2(a) and 2(b) indicate that the promoting effects from the punishments and individuals' sensitivity have an upper limit, because the attractive range for full punishment (FP), i.e., the system is full of punishers, identified by the position of the intermediate state, remains almost invariable. A negative valley in terms of Δ_p becomes visible when the parameter condition is not so desirable for the punishers (i.e., r, α, and A are small), suggesting that CP is more effective than DP in suppressing defectors under well-mixed conditions. As the three parameters become larger, there is no significant difference between the two types of punishment with respect to promoting public goods; as a result, Δ_p is negative but very close to zero. Nevertheless, none of the punishers can occupy a superior position facing defectors, which is confirmed by the phenomena $p_{\mathrm{CP}} > 0.5$ and $p_{\mathrm{DP}} > 0.5$ in Figs. 2(a) and 2(b), respectively.

Next we shift our attention to the three-strategy situations TC + D + CP, TC + D + DP, and TC + CP + DP. Figure 3 reveals the monotonic effects of facilitating the advantages of nondefectors of the synergy effect, punishment fine, and feedback sensitivity, through presenting the ratios of attraction basins of SP under various parameter conditions (see Figs. 16–18 and the corresponding descriptions for more detailed information on attraction basin patterns for the three-strategy situations). In contrast, an obvious saturation effect can be found in the D + CP + DP case, where the ratios reach a limited intermediate value with increasing punishment fine or feedback sensitivity [see the middle and right panels in Fig. 3(c)]. In addition,, comparatively higher ratios suggest that a mix of CP and DP works better in sustaining the public goods without involvement of second-order free riders, i.e., traditional cooperators, which is in accordance with the conclusion from the empirical studies that a combination of self-regulation and government oversight lead to a better performance in improving market tracking [33,46]. On the other hand, it can be noticed in the cases with only punishment that the performance of the synergy effect goes through a sharp transition from being unimpressive to being rather remarkable near the point $r = G$, above which nondefectors dominate while all defectors die out, which is more or less in agreement with the findings of previous works [71].

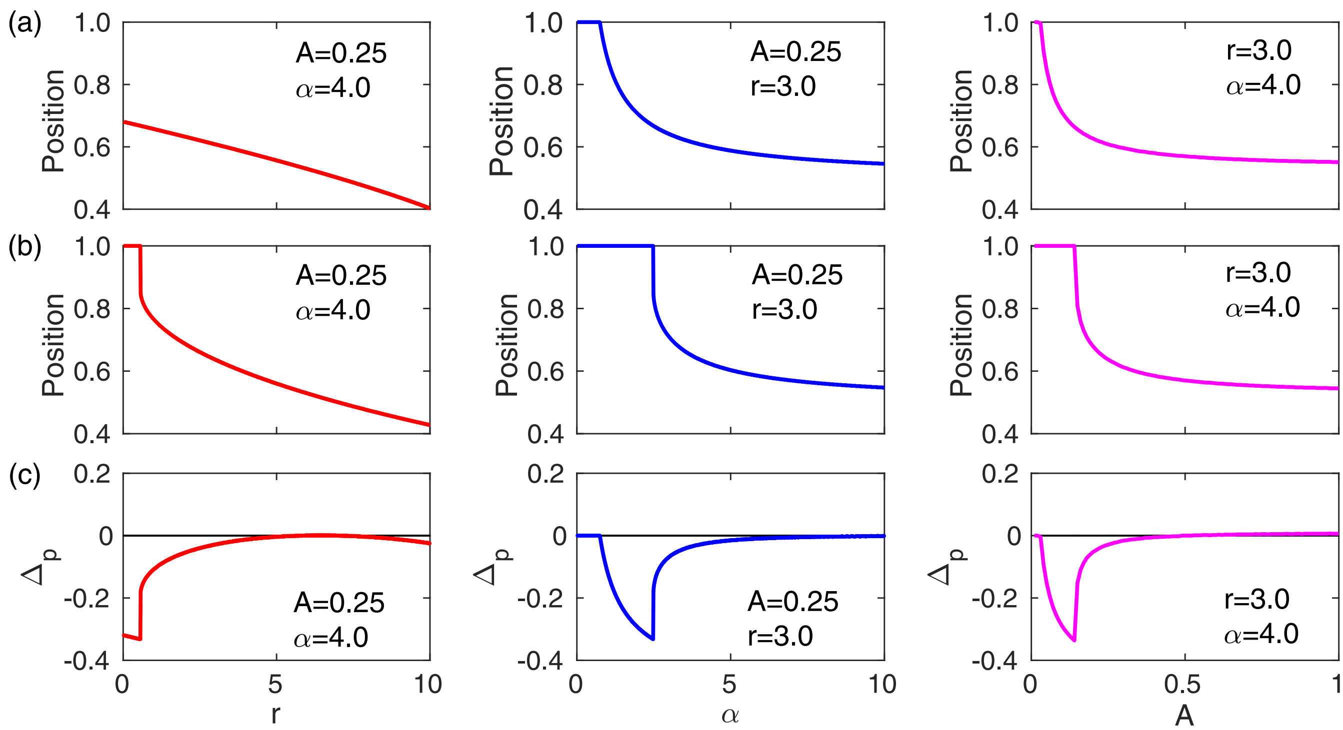

FIG. 2. Illustrations of the position of the coexistence state of two strategies under various parameter conditions for (a) the D + CP case and (b) the D + DP case. Specific values of the other parameters are given in each legend. The position value p_{CP} (p_{DP}) indicates the position of intermediate unstable state (i.e., the open circles illustrated in Figs. 14 and 15 of Appendix B) in the D + CP (D + DP) case, which can be used to estimate the advantages of the punishment strategy. More specifically, the smaller p_{CP} or p_{DP} is, the more likely the system would be to reach the state of full punishment. Also illustrated is (c) the difference $\Delta_p = p_{\mathrm{CP}} - p_{\mathrm{DP}}$ between two position values so that we can judge which punishment is more effective in enlarging the attractive range of FP, i.e., promoting public cooperation.

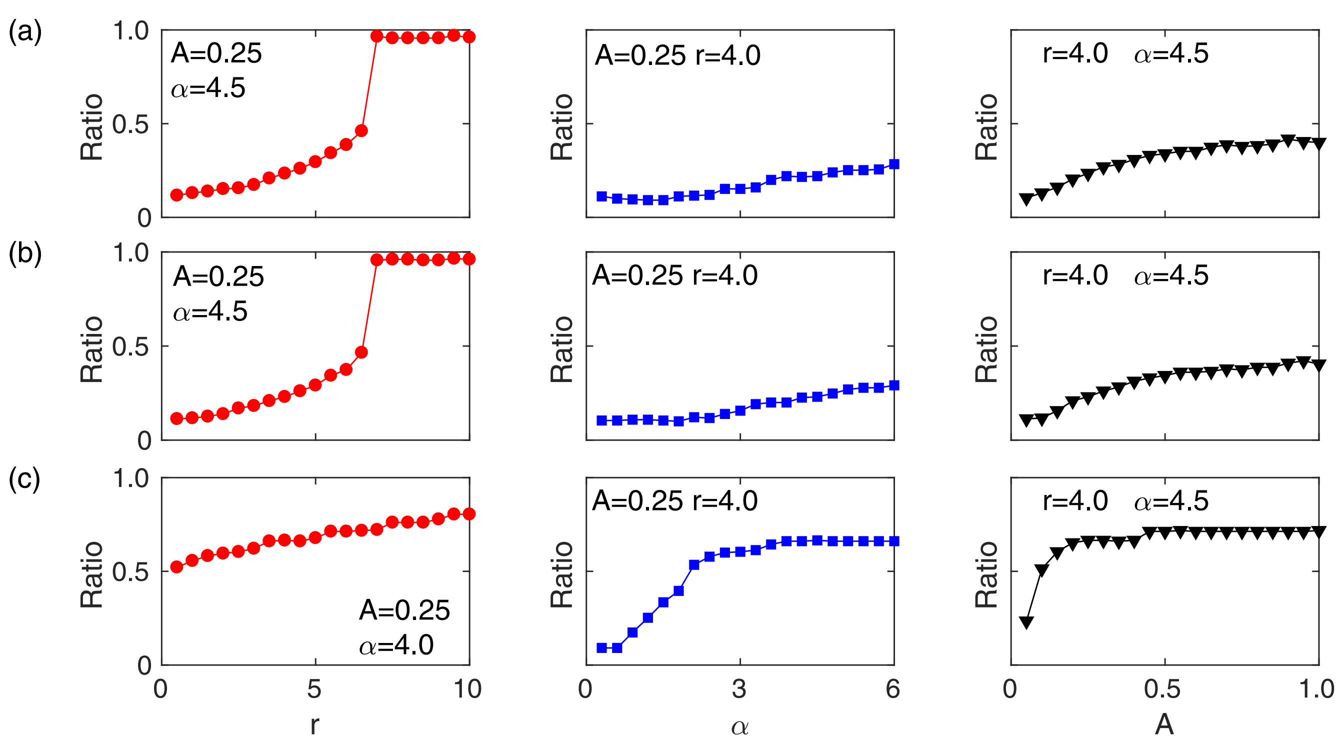

FIG. 3. Ratios of the areas of the attraction basins of the SP state versus r, α, or A for three different evolution situations: (a) TC + D + CP, (b) TC + D + DP, and (c) D + CP + DP.

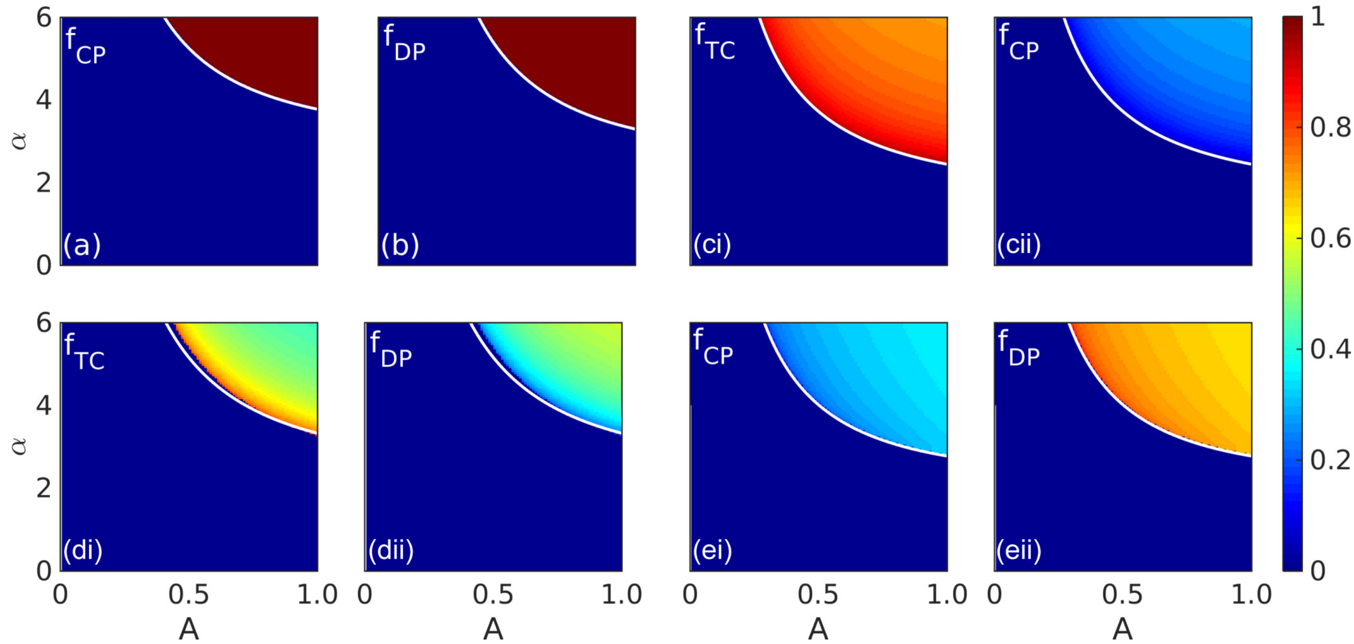

FIG. 4. Analytical dependence of the final steady fractions of different nondefective strategies on both A and α for different evolutionary situations: (a) D + CP, (b) D + DP, (c) TC + D + CP, (d) TC + D + DP, and (e) D + CP + DP. The initial conditions for each case are (a) $f_{\rm CP}(0) = 0.54$, (b) $f_{\rm DP}(0) = 0.54$, (c) $f_{\rm CP}(0) = 0.45$ and $f_{\rm TC}(0) = 0.22$, (d) $f_{\rm DP}(0) = 0.53$ and $f_{\rm TC}(0) = 0.05$, and (e) $f_{\rm CP}(0) = 0.3$ and $f_{\rm DP}(0) = 0.25$. Correspondingly, the values of f_s used to semianalytically estimate the boundary lines are (see Appendix C for further details) (a) $f_{\rm CP} = 0.54$, (b) $f_{\rm DP} = 0.54$, (c) $f_{\rm TC} = 0.22$ and $f_{\rm CP} = 0.356$, (d) $f_{\rm TC} = 0.05$ and $f_{\rm DP} = 0.492$, and (e) $f_{\rm CP} = 0.3$ and $f_{\rm DP} = 0.25$. In all cases, the value of the synergy factor is $r = 4.0$.

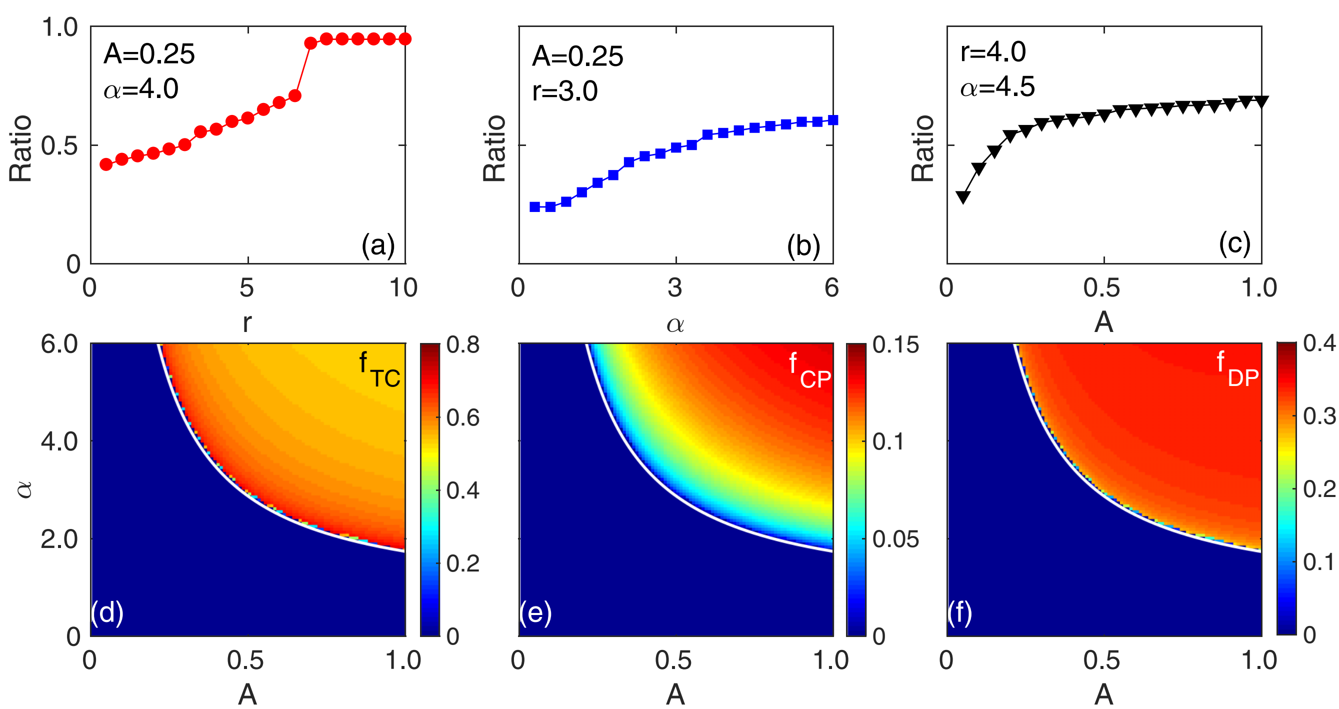

FIG. 5. Comprehensive understanding of the evolutionary dynamics for the evolutionary situation TC + D + CP + DP. (a)–(c) Volume ratios of the attraction basins of the SP state versus r, α, or A, with the values of other parameters given in each legend. (d)–(f) Analytical dependence of the final steady fractions of three nondefection strategies on both A and α, along with the semianalytical solid boundary lines. The initial condition is $\rho_S(0) = 0.25$ ($S \in$ {TC, D, CP, DP}). Correspondingly, the values of f_s used to semianalytically estimate the boundary line are $f_{\rm TC} = f_{\rm DP} = f_{\rm CP} = 0.25$. The other parameter is $r = 4.0$.

Furthermore, Figs. 4 and 5 together provide a comprehensive picture of strategy fractions in the parameter plane (A, α) for six different evolution situations, as well as semianalytically estimated boundary lines and volume ratios of attraction basins of segment punishment (SP) (the state where punishment is found to coexist with other strategies) for the case $\mathrm{TC} + \mathrm{D} + \mathrm{CP} + \mathrm{DP}$. In particular, f_{TC}, f_{CP}, and f_{DP} represent the final steady fractions of traditional cooperators, cooperator-driven punishers, and defector-driven punishers in the population, respectively. Most obviously, the regions of the nondefection (ND) phase are found to be in the top-right corner, which can be considered as support for the observed monotonic effects of facilitating the advantages of nondefectors of r, α, and A from another perspective, since defectors bear a higher cost of punishment, induced by both more frequent punishments and the higher cost of one punishment. It can also be expected that the semianalytical approach relying on the well-mixed assumption can give "perfect" boundaries to distinguish SP phases from FP phases. Cooperator-driven punishers are in a disadvantaged position compared to traditional cooperators or even defector-driven punishers [more traditional cooperators or defector-driven punishers exist in the final state; see Figs. 4(c), 4(e), and 5(d)–5(f)]. Further support for requiring a nontrivial interplay between defector-driven and cooperator-driven punishers in promoting the public cooperation is also verified in Fig. 4(e), where a larger region of the ND phase exists. At this point, CP is frequently enforced when nondefectors dominate the population, while DP efficiently works on the condition that defectors become the majority. Therefore, sufficiently available punishments are always provided in spite of the abundance of strategies. In particular, through an in depth comparison of the illustrations in Fig. 3 with the ratios in Fig. 5, we can say that the transition in terms of the ratio is mainly due to synergistic effects from cooperation, instead of punishments themselves. In other words, punishment fines and feedback sensitivity are the parameters that mainly govern the performance of punishment measures in an infinite well-mixed population. Finally, combining Figs. 3 and 5, we find that traditional cooperators are traditionally held responsible for preventing the dominance of cooperator-driven and defector-driven punishers, in accordance with conclusions from previous studies [10–12].

B. Networked populations

The study of the present model under the infinite well-mixed condition reveals the monotonic effects of synergistic effects, punishment fines, and feedback sensitivity in facilitating public cooperation. In accordance with the conclusions from few previous empirical studies involving internet coregulation [74], the combined action of CP and DP gives a better performance. In addition, a non-negligible deviation from reality is that CP is always outperformed by DP or TC. However, in reality, interactions among individuals are not typically random but rather highly structured, i.e., each individual has a fixed neighborhood to some extent [75–77]. Taking this realistic factor into consideration, we find some counterintuitive results not usually uncovered in a well-mixed population.

Figures 6 and 7 together exhibit what role feedback sensitivity plays in governing the evolution dynamics in six different evolutionary situations. Of particular interest is that an optimal intermediate range of sensitivity A for prevalence or even complete dominance of punishers can be found under suitable parameter conditions in each situation. This is definitely different from what happens in well-mixed populations. More specifically, in the case that cooperator-driven punishers face defectors alone, the peaks of f_{CP} become wider until a threshold of punishment fine above which they exhibit an monotonic increase [see Fig. 6(a)]. After the introduction of traditional cooperators, more sharp peaks of f_{CP} can be observed for large α, along with a shift of these peaks toward smaller A with increasing punishment fine [see Fig. 6(cii)]. In contrast, the emergence of optimal intermediate ranges of A for dominance of defector-driven punishers is instead facilitated by large punishment fines for which either of two peaks of f_{DP} is present when traditional cooperators are additionally introduced [see Figs. 6(b) and 6(dii)]. In the cases considering only punishment, the positions of peaks remain more or less independent of whether TC intervenes. However, the optimal intermediate ranges of feedback sensitivity A would shrink (see Fig. 6), which still holds in the cases with a mix of punishments [see Figs. 6(e), 7(b), and 7(c)]. This arises from the fact that punishers would suffer from second-order free-riding behaviors, which also contributes to a monotonic increase of traditional cooperators with the growth of punishers [Figs. 6(ci), 6(di), and 7(a)]. Additionally, a key hint at the competitive relationship between the two types of punishers is also revealed by the results presented in Fig. 6(e) that positions of population peaks of DP correspond rightly to the valleys of CP population when that punishment fine is large enough. Figures 6(e) and 7(b) also show that stronger punishment indicated by higher α does not always mean higher achievable levels of CP in the population as both punishment measures are taken together. There is also an optimal intermediate range of α for the prevalence of CP individuals. In the presence of all four strategies, peaks of f_{CP} rather than f_{DP} still occur for large α [Figs. 7(b) and 7(c)]. Finally, by comparing Fig. 6(a) with Fig. 6(b) or Figs. 6(c) and 6(d), we note that CP is more efficient than DP with respect to maintaining public goods, by promoting a wider range of A for prevalence of punishers with smaller punishment fines.

The microscopic mechanism behind the reported optimal intermediate feedback sensitivity in the $\mathrm{D} + \mathrm{CP}$ case is rather revealed by the behaviors of different key statistical characteristic quantities presented in Fig. 8. In particular, the spectrum of payoff gaps $\Delta_{\mathrm{CP\text{-}D}}(n_{\mathrm{CP}}, n_{\mathrm{D}})$ and different states of edges $E_{\mathrm{CP\text{-}D}}(n_{\mathrm{CP}}, n_{\mathrm{D}})$ presented in Fig. 8 reveal more detailed information about the spatial pattern formations. When A is small, unresponsive CP individuals would exert too few punishments on defectors within the groups, leaving expanded opportunities for defectors. The fraction of cooperator-driven punishers thus decreases to zero. Conversely, for large A cooperator-driven punishers are too sensitive to punish too many defectors of different groups, which could greatly reduce the punishers' payoffs (see the top right panel of Fig. 8), especially those with fewer than three connected punishers (Fig. 8), and further shrink their formed islands. This implies that optimal feedback sensitivity to maximize

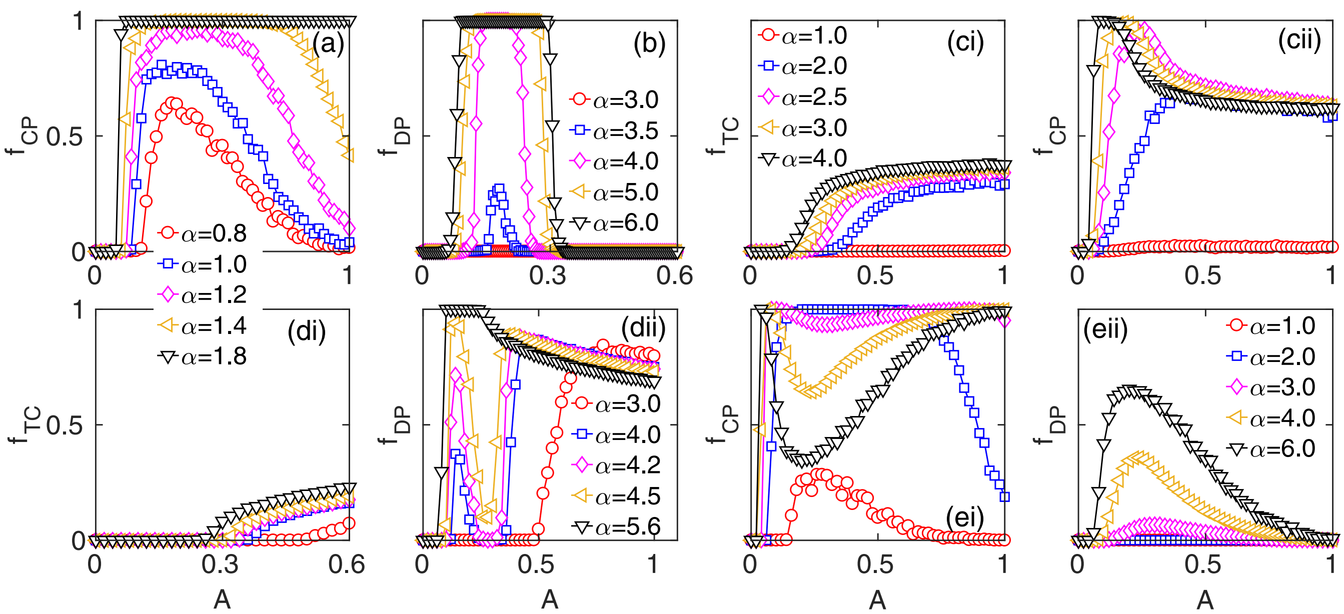

FIG. 6. Final steady fractions of different nondefective strategies as a function of A for networked populations; the results for five different evolution situations are illustrated: (a) D + CP, (b) D + DP, (c) TC + D + CP, (d) TC + D + DP, and (e) D + CP + DP. In all cases, $r = 4.0$.

CP population should be intermediate. Under such parameter conditions, cooperator-driven punishers could not only defeat defectors through sufficient and strong punishment (large α) but also maintain competitive advantages (especially those having fewer than three neighbors of the same strategy; see the middle column in Fig. 8) at the borders of CP clusters so as to finally expand permanently by absorbing defectors. In particular, the isolated cooperator-driven punishers near the clusters or those at the tip of peninsulas located at the borders of clusters are pioneers of expansions. The indispensable role of the two types of pioneers for CP clusters is revealed by the spectrum illustrated in Fig. 8 in which one can find that whether the punishers possessing fewer than three connected punishers have higher payoffs than those defective neighbors with the same neighborhood state is mainly responsible for the final dominance of punishers. As a result, there are considerable long-standing edges of corresponding states for optimal values of A (the red patterns shown in middle panel of the middle column in Fig. 8). A similar mechanism leading to the dominance of defector-driven punishers in the D + DP case is also revealed by the illustrations in Fig. 19 in Appendix B. Nevertheless, both types of punishers are inclined to clustering because of their prosocial nature, i.e., adopting cooperation strategy before exerting punishment. However, a larger punishment fine is required as a remedy to weak network reciprocity caused by defector-driven punishers.

The observed behaviors in Figs. 8 and 19 provide a refined physical picture of the clustering behaviors of punishers at two distinctly different stages. (1) At the precluster stage, owing to different sources to drive punishment executions, cooperator-driven punishers prefer to reduce punishment so as to preserve more competitive payoffs, while defector-driven punishers have limited payoffs resulting from more frequent executions. Meanwhile, support from network reciprocity is lacking, because compact clusters have not yet formed so early in the process. It turns out that cooperator-driven punishers are more likely to persevere to organize themselves into clusters, and therefore the required minimum size for their

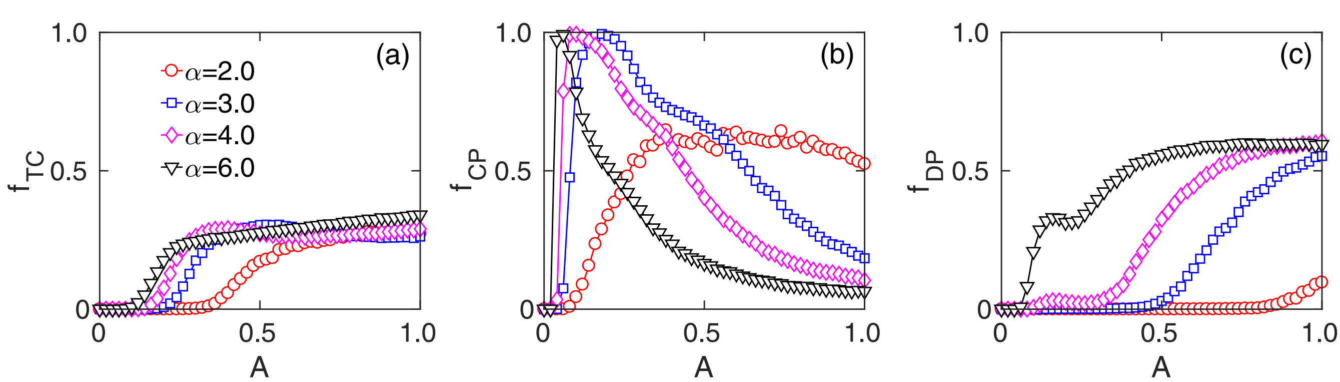

FIG. 7. Final steady fractions of different nondefective strategies as a function of A for networked populations for four strategies: TC, D, CP, and DP. The value of the synergy factor is $r = 4.0$.

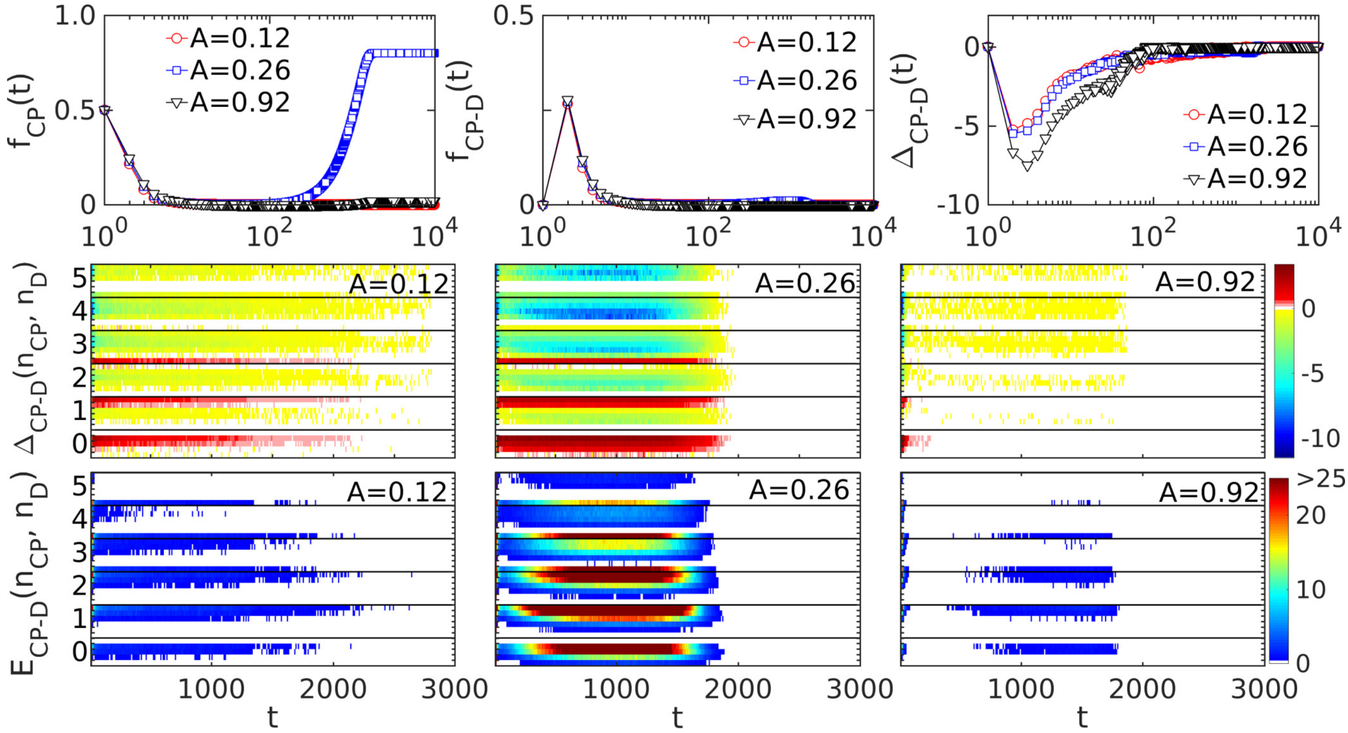

FIG. 8. Evolutionary dynamics of the networked populations for three representative values of A, (a) $A = 0.14$, (b) $A = 0.26$, and (c) $A = 0.92$, in the D + CP case. Five key statistical characteristic quantities are shown for $r = 4.0$ and $\alpha = 1.0$: the fractions of cooperator-driven punishers $f_{\text{CP}}(t)$ (top left panel); the fractions of edges connecting the punishers to defectors $f_{\text{CP-D}}(t)$ (top middle panel) which are normalized by the total number of edges in the networks; the mean payoff gap between a cooperator-driven punisher and its connected defector, where an imitation process happens $\Delta_{\text{CP-D}}(t) = \sum^{E'_{\text{CP-D}}(t)}(\Pi_{\text{CP}} - \Pi_D)/E'_{\text{CP-D}}(t)$ (top right panel) [$E'_{\text{CP-D}}(t)$ represents the total number of occurrences of the imitation process among CPs and Ds at time t]; the payoff-gap spectrum [$\Delta_{\text{CP-D}}(n_{\text{CP}}, n_{\text{D}})$] of $\Delta_{\text{CP-D}}(t)$ for 36 different neighborhood states, which are determined by the number of cooperator-driven punishers that can be found among their neighbors (the middle panels), where n_{CP} (n_{D}) represents the number of cooperator-driven punishers that can be found among the neighbors of a cooperator-driven punisher (defector); the number spectrum of 36 different states of edges $E_{\text{CP-D}}(n_{\text{CP}}, n_{\text{D}})$ whose states are determined by the number of cooperator-driven punishers each of the two ends have. In detail, the six numbers marked on the y axis indicate the possible number of neighbors who are found to be cooperator-driven punishers for a cooperator-driven punisher. There are thus six scales for each number (grid box), each of which indicates, respectively, the number of cooperator-driven punishers the defective neighbor has (from bottom to top, the number of punitive neighbors of this defector is 0–5), rising to 36 scales in total.

growth of clusters is smaller than that of defector-driven punishers. (2) At the postcluster stage, defector-driven punishers become instead conservative in punishing defectors. At this point, cooperator-driven punishers have strong support from the formed clusters on the one hand, and on the other hand there are enough sources to drive them to provide a sufficiently effective punishment. To sum up, in the two-strategy cases, cooperator-driven punishers are superior to defector-driven individuals in terms of both taking advantage of network reciprocity and suppressing defectors.

Figure 9 gives the quantitative traits of representative spatial evolution of the three competing strategies, TC, D, and CP, for parameter values that yield different absorbing phases. Remarkably, the additional participation of traditional cooperators can produce a somewhat different evolution picture in which cooperator-driven punishers can possibly outperform defectors to leave survival spaces for traditional cooperators and further form a stable coexistence with them. When A is small, unresponsive cooperator-driven punishers are naturally defeated by defectors and finally made extinct, along with the disappearance of TC. As the sensitivity A increases, cooperator-driven punishers begin to conquer the whole network due to the same mechanism revealed in the two-strategy cases. This increases the positive peak of $f_{\text{CP-D}}(t)$ (the fractions of edge CP-D) as well as $\Delta n_{\text{CP-D}}(t)$ (net increase of cooperator-driven punishers) (see Fig. 9). As the sensitivity is increased further, slightly positive $f_{\text{CP-D}}(t)$ and larger positive peaks of $f_{\text{TC-D}}(t)$ can be observed, indicated by pink markers in Fig. 9. In such cases, we have revealed a strong mutualism between TC and CP, single-strategy clusters of which cannot persist in the sea of defectors. Also, traditional cooperators at the borders of CP clusters actually play the role of a "protective film," which spatially isolates the punishers bearing additional punishment cost from those defectors, and thus prevents them from being eroded. This in turn gives these traditional cooperators, i.e., second-order free riders, advantaged position (slightly positive $\Delta_{\text{TC-D}}$ indicated by pink and black markers in Fig. 9) to outperform their defective neighbors whose payoffs have been greatly reduced by punishers within the same game groups. That is why $f_{\text{TC-D}}(t)$ and $f_{\text{CP-TC}}(t)$ are obviously positive while $f_{\text{CP-D}}(t)$ is approximately zero (the second row in Fig. 9). Meanwhile, the dynamics is formulated by the majoritylike rule in the areas far from the borders of the clusters, because there is no

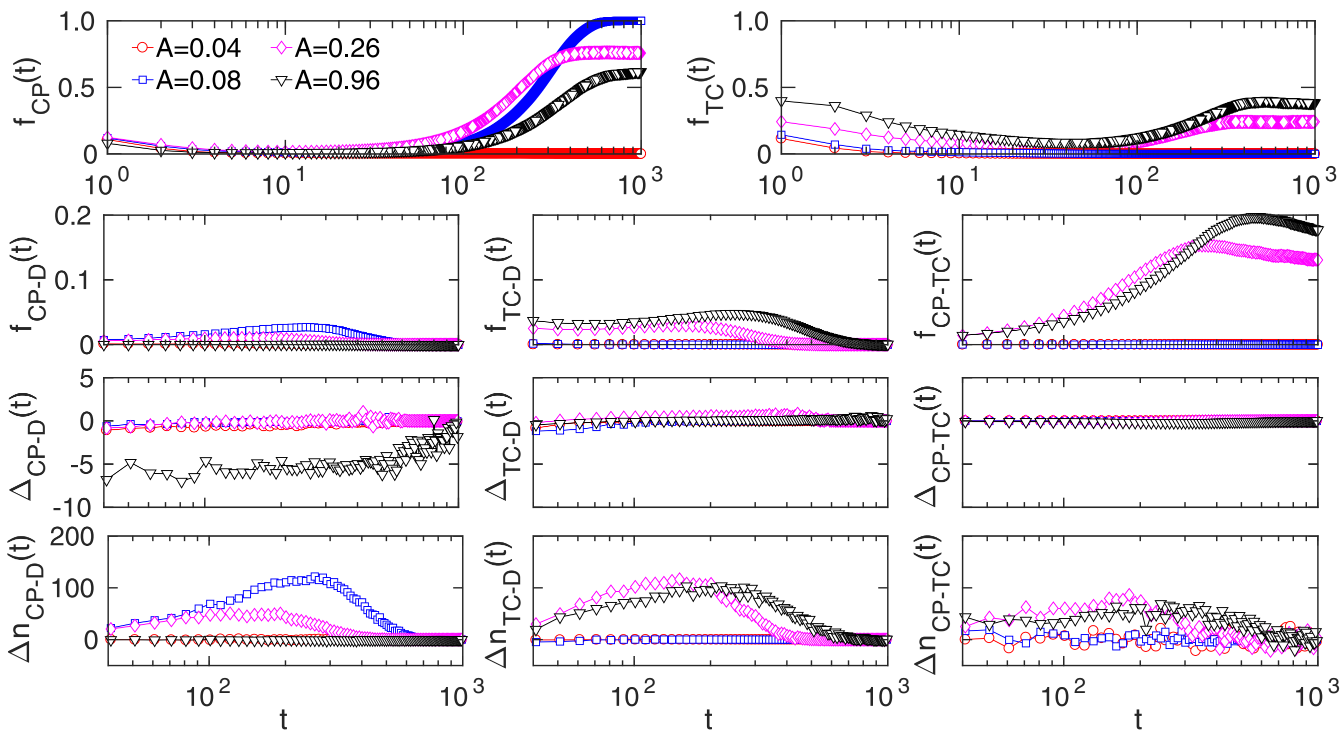

FIG. 9. Dynamic changes of statistical characteristic quantities for four different representative values of A in networked populations. In detail, from the top row to the bottom, these statistical statistical quantities are the fractions of cooperator-driven punishers $f_{\text{CP}}(t)$ [traditional cooperators, $f_{\text{TC}}(t)$] in the population; the fractions of three different edges (CP-D, TC-D, and CP-TC) which are normalized by the total number of edges in the population (middle panels), where $f_{\text{CP-D}}(t)$, $f_{\text{TC-D}}(t)$, and $f_{\text{CP-TC}}(t)$ denote the fractions of the corresponding edges, respectively; the mean payoff gaps between the two ends of an edge at which an imitation process between different strategies happens, which is defined in the same manner proposed in the caption of Fig. 8; and the net increase of cooperator-driven punishers (traditional cooperators) who are produced from the imitation process between defectors (traditional cooperators) and cooperator-driven punishers or traditional cooperators (cooperator-driven punishers) at time t (bottom panels), e.g., $\Delta n_{\text{CP-D}}(t) = n_{\text{D}\to\text{CP}}(t) - n_{\text{CP}\to\text{D}}(t)$, where $n_{\text{D}\to\text{CP}}(t)$ [$n_{\text{CP}\to\text{D}}(t)$] indicates the number of occurrences of the imitation process which successfully translates a defector (cooperator-driven punisher) into a cooperator-driven punisher (defector) at time t. The other parameters are $r = 4.0$ and $\alpha = 4.0$.

difference between TC and CP individuals in the absence of defectors. Correspondingly, there is a large number of CP-TC edges of which the two ends have equal payoffs [$\Delta_{\text{CP-TC}}(t)$ is approximately zero from beginning to end]; however, positive $\Delta n_{\text{CP-TC}}(t)$ reveals a considerable translation from TC to CP (the bottom row in Fig. 9).

The system also presents dynamic traits similar to those in the TC + D + DP case, as shown in Figs. 20 and 21 in Appendix B. However, it is worth noting that DP clusters are not as strong as CP clusters in terms of resisting defectors, owing to the fact that defector-driven punishers become unresponsive as they cluster (as punishment-driven sources, defectors within the same game groups become fewer in number). As another consequence, DP clusters are less favorable for the survival of surrounding traditional cooperators, and these cooperators are more dependent on network reciprocity to form more rounded clusters (the fourth and fifth rows in Fig. 21). Thinner layers consisting of small TC clusters can also be found (see Fig. 21).

We have checked that the results for the evolutionary situation D + CP + DP are consistent with the illustrations in Fig. 6(e) that cooperator-driven punishers are prior to defector-driven ones. The results reveal that the cooperator-driven punishers' prevalence depends mainly on their success in the battle against defectors. Furthermore, cooperator-driven punishers seem more essential for the survival of defector-driven punishers since DP clusters cannot persist and they have to combine with CP clusters who can fully take advantage of network reciprocity (for more details, see Fig. 22, along with the corresponding descriptions).

Given the condition that all four strategies are present, one can expect the following evolutionary patterns of different strategy clusters shown in Fig. 10, based on the mechanisms revealed for three-strategy cases: Unlike TC and DP clusters, CP clusters can exist alone in the face of defectors, while DP or TC clusters have to combine with each other or with CP clusters.The DP or CP clusters are more or less enfolded by traditional cooperators for high sensitivity A, indicating the establishment of an additional mutualism between traditional cooperators and the two types of punishers. The majority-like rule still formulates the dynamics in the interiors of nondefectors' clusters where there is no essential distinction between punishers and traditional cooperators, which can be considered as the key mechanism behind the competitive relationship between the two types of punishers, as also shown in Fig. 6. Cooperator-driven punishment is superior to TC and DP for a wide range of the sensitivity A, especially when the parameter is intermediate. Furthermore, Fig. 6 reveals that the

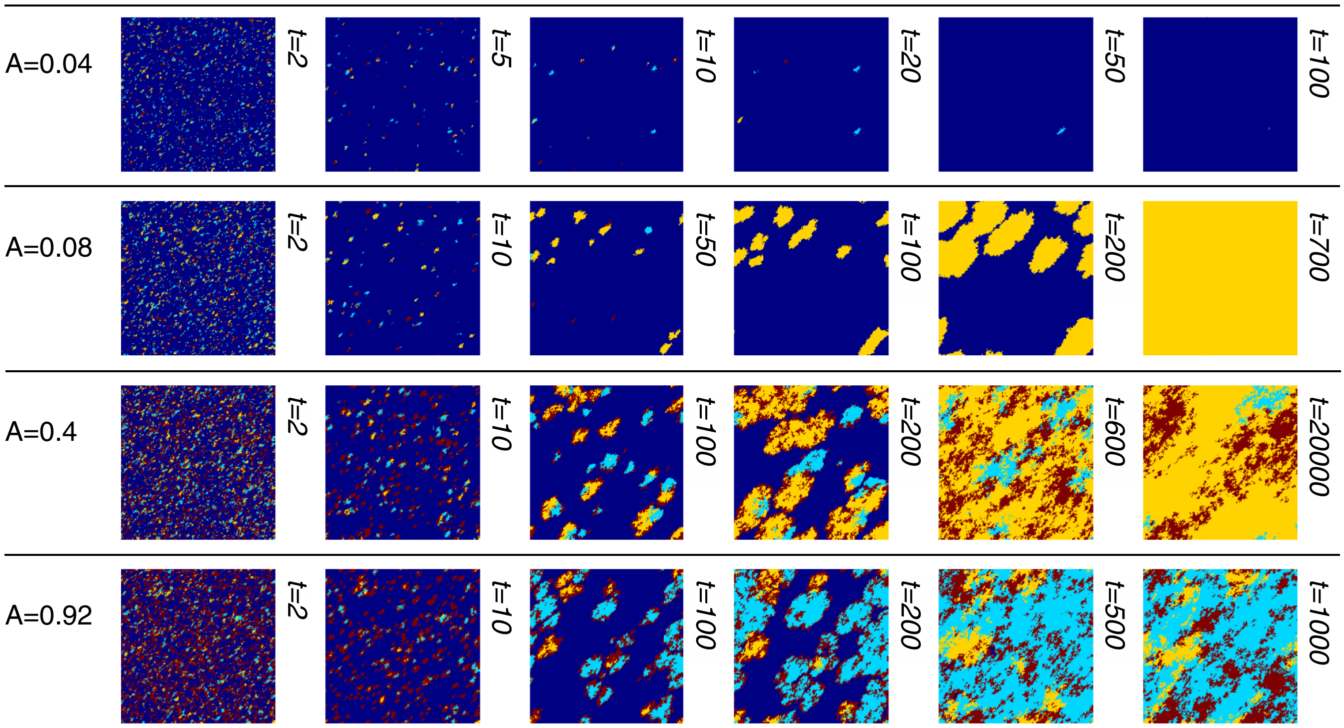

FIG. 10. Representative spatial evolution of the four strategies TC, D, CP, and DP in networked populations for four representative values of A. Depicted are snapshots of the hexagonal lattice with size $L = 200$, where the punishment fine is $\alpha = 4.0$. Cooperators (defectors) are shown in maroon (dark blue), while cooperator-driven (defector-driven) punishers are depicted in yellow (light blue).

above-mentioned microscopic mechanisms for three-strategy cases do not rely on any additional strategic complexity limiting their general validity.

Since stochastic imitation of a neighboring strategy ensures the existence of (homogeneous) absorbing states and critical phase transitions, we have also explicitly explored the dynamic behaviors of the system in the close vicinity of transition points above which the system shifts from the full-deflection (FD) phase to the FP phase. In the case of D + CP, it can be observed in Fig. 11(a) that in the close vicinity of transition point, the final outcome in the two cases is remarkably different while the difference in the sensitivity A is minute, which is a characteristic feature of discontinuous phase transition. Moreover, similar but more obvious discontinuous behaviors in terms of the fractions of either DPs or CPs can be produced in the cases of TC + D + DP, D + CP + DP, and TC + D + CP + DP [see Figs. 11(b)–11(d)] and other evolutionary situations. This is in accordance with what has been uncovered from the previous studies involving pool punishment against defectors [12].

Finally, Figs. 12 and 13 provide a comprehensive picture in the parameter plane (A, α) of the evolutionary dynamics, as well as semianalytically estimated boundary lines (see Appendix C for more details). The boundaries between the regions of different nondefective strategies are not given. By means of the results displayed in Fig. 11, note that a discontinuous phase transition always occurs when the system shifts from the FD phase to the FP phase, instead of a continuous phase transition from the FD phase to the SP phase, irrespective of the complexity of evolutionary situations. The discontinuous phase transition is due to the positive feedback arising from the fact that moderately sensitive punishers first begin to cluster and then keep growing to completely dominate the whole population along with the extinction of traditional cooperators, while the continuous phase transition stems from increasing competitiveness of punishers strengthened by punishment α [73] and persistence of traditional cooperators. Overall, CP can more effectively enforce a regulation to promote and sustain public goods through enlarging the regions of the ND phase, and thus be more prompt than DP, especially in the absence of traditional cooperators. More specifically, the performances of cooperator-driven punishers can be weakened by free-riding behaviors of traditional cooperators [see Figs. 12(a) and 12(c)] who may however greatly help defector-driven punishers to beat defectors in a much larger parameter space [see Figs. 12(b) and 12(d)]. In both cases, two types of punishers in turn provide survival space for traditional cooperators, and hence regions of TC more or less coincide the those of the DP or CP phase [see Figs. 12(c) and 12(d)], but not totally. In the presence of more than two strategies, the optimal parameter regions for different nondefective strategies repel each other, especially between the two punishing strategies, as a result of the majoritylike rule. Meanwhile, this rule rises to another notable result that performance of CP is more or less limited by DP. Also, nonmonotonic changes of alternating frequencies of both punishing strategies are more clearly illustrated, which is a consequence of both network reciprocity and the majoritylike rule among nondefective individuals. Finally, notice that the semianalytic estimated boundaries successfully distinguish the simulated regions of NP phases from the whole parameter space, although there is deviation in the case of D + DP for

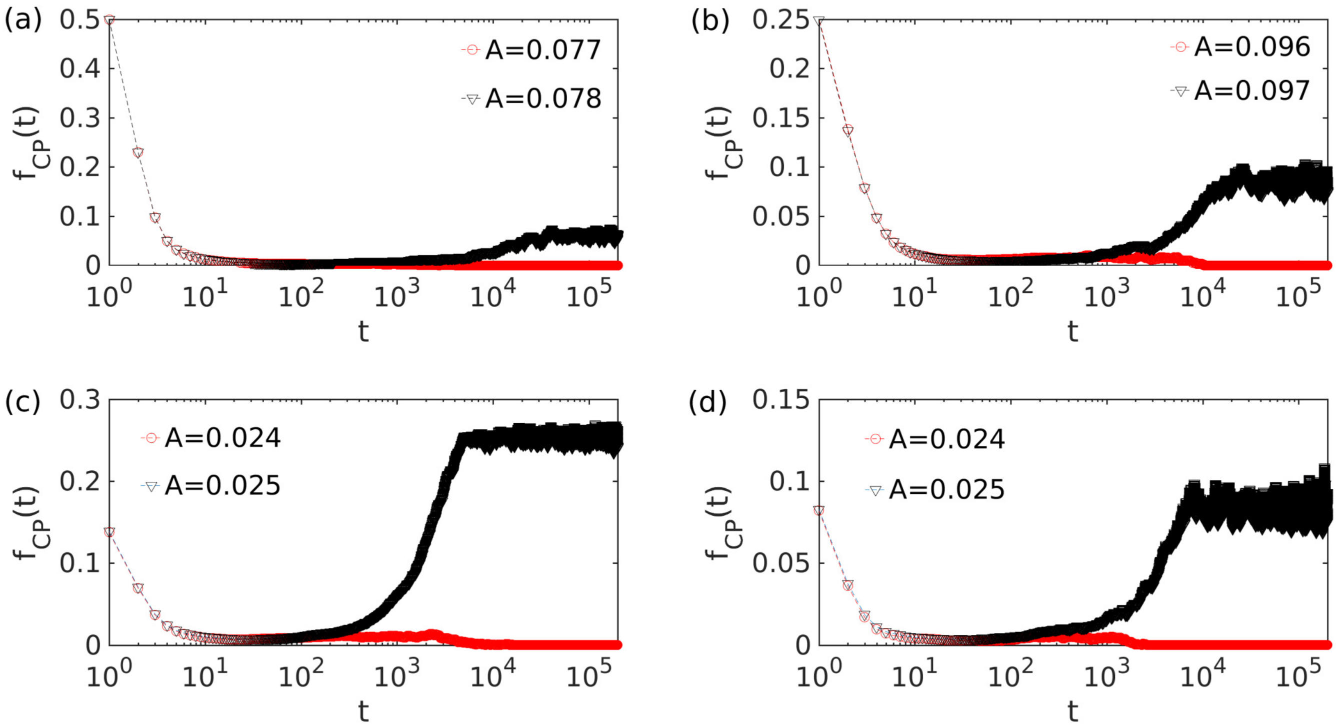

FIG. 11. Evolution of the fractions of cooperator-driven punishers over time, starting with a random initial state. The results are presented for four different evolutionary situations, (a) D + CP, (b) TC + D + DP, (c) D + CP + DP, and (d) TC + D + CP + DP, in the immediate vicinity of transition points. The values of the punishment fine are (a) $\alpha = 1.8$, (b) $\alpha = 4.4$, (b) $\alpha = 6.0$, and (d) $\alpha = 6.0$, respectively. The other parameter is $r = 4.0$.

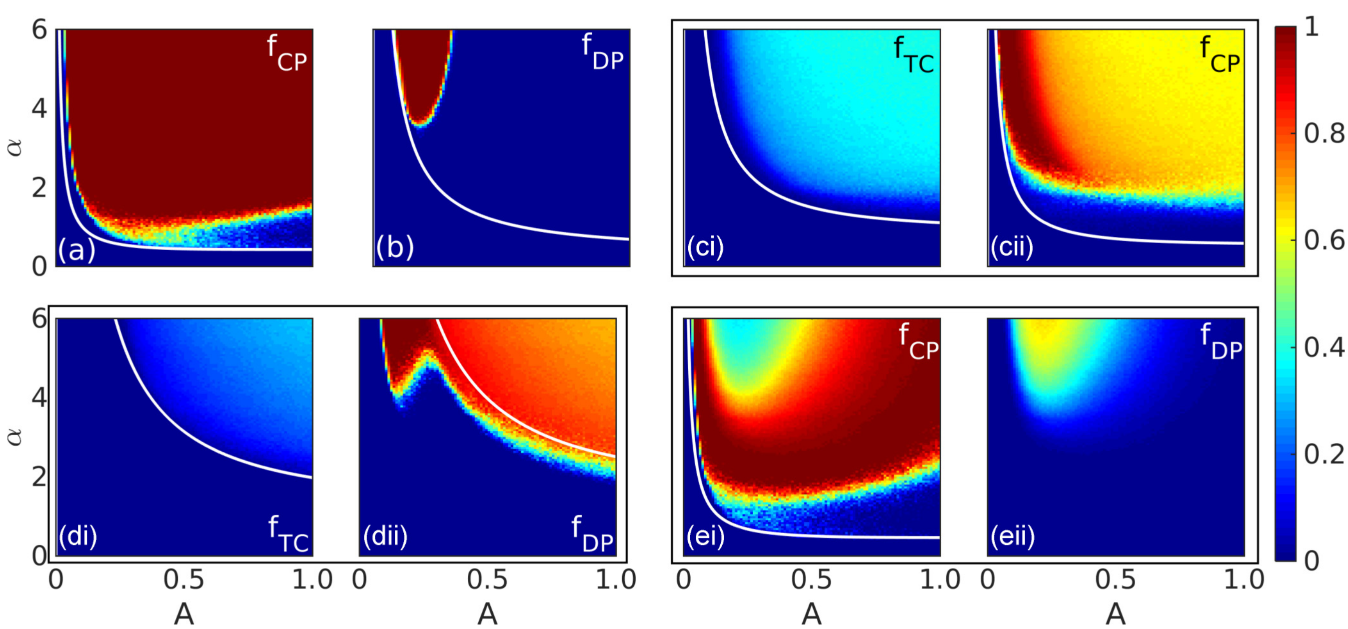

FIG. 12. Dependence of simulated final steady fractions of different nondefection strategies on both A and α, for five different evolution situations: (a) D + CP, (b) D + DP, (c) TC + D + CP, (d) TC + D + DP, and (e) D + CP + DP. The population is networked and $r = 4.0$. Correspondingly, the value of f_s used to semianalytically estimate the boundary lines (see Appendix C for further details): (a) $f_{CP} = 1.0$, (b) $f_{DP} = 0.96$, (c i) $f_{TC} = 0.1$ and $f_{CP} = 0.55$, (c ii) $f_{TC} = 0.1$ and $f_{CP} = 0.71$, (d i) $f_{TC} = 0.115$ and $f_{DP} = 0.465$, (d ii) $f_{TC} = 0.1$ and $f_{DP} = 0.46$, (e i) $f_{CP} = 0.1$ and $f_{DP} = 0.84$, (e ii) $f_{CP} = 0$ and $f_{DP} = 0.95$.

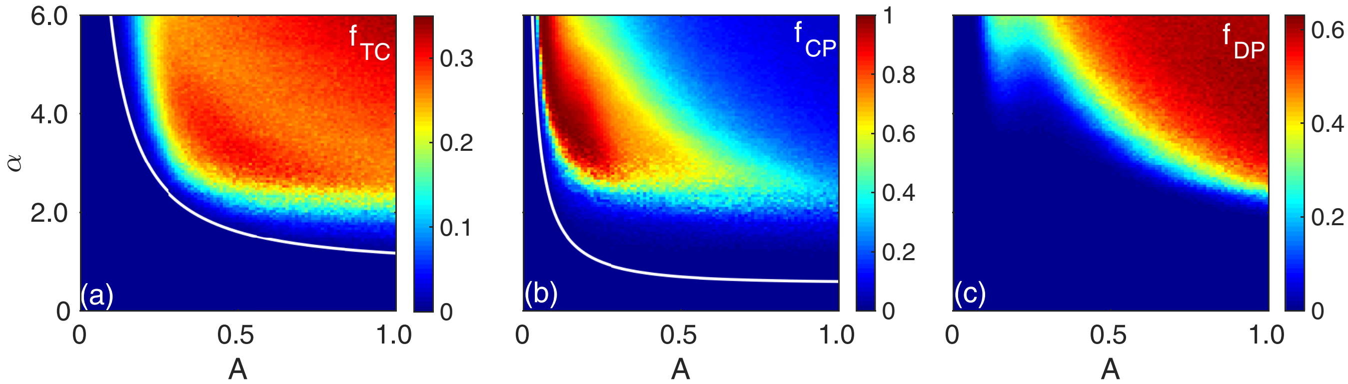

FIG. 13. Dependence of simulated final steady fractions of three different nondefection strategies on both A and α. The population is networked, with four strategies TC, D, CP, and DP considered. Correspondingly, the values of f_s used to semianalytically estimate the boundary line are (a) $f_{\rm TC} = 0.09$, $f_{\rm DP} = 0.09$, and $f_{\rm CP} = 0.46$ and (b) $f_{\rm TC} = 0$, $f_{\rm DP} = 0$, and $f_{\rm CP} = 0.8$. The other parameter is $r = 4.0$.

large A [Fig. 13(b)]. This discrepancy is attributed to a vicious circle between defectors and defector-driven punishers: The more defectors the punishers punish, the lower payoffs they have, and thus more of them go extinct. Finally, there are more defectors to further drive the punishers to exert punishments with a higher probability. Consequently, defector-driven punishers go extinct in a much larger parameter region than theoretically expected based on well-mixed situation, which is exacerbated in the ranges of large A.

By comparing the results shown in Figs. 23 and 13 with corresponding analytical predictions given by Figs. 4 and 5 with the initial conditions unchanged, we can conclude that the networked structure of the population is essential for the superior performance of cooperator-driven punishers to defector-driven ones. Hence the system can achieve a more desirable level of public goods.

IV. CONCLUSION

To summarize, empirical explorations of corporate self-regulation and government regulation suggest two different punishment measures which play indispensable roles in regulatory issues, calling for a general game model to take into account both types of punishments. We have accomplished that in this paper. For the purpose of fully identifying both the interactions among different strategies and the performances of the two punishments, we have considered six different evolutionary situations with and without traditional cooperation, in which one or both punishment strategies are introduced to fight against defectors. In addition, we have proposed a theoretical approach to completely describe the evolutionary dynamics of the six different combinations of strategies in an infinite well-mixed population. Agent-based simulations were employed to give numerical results for networked populations embedded on a regular lattice. At the same time, we have developed a semianalytical method which allows us to give relatively accurate estimations of the boundaries between full-defection and nondefection phases in most evolutionary situations.

For the infinite well-mixed population, we first used a series of replicator equations capturing features of the present model to give gradients of selection for two-strategy cases, phase portraits for three-strategy cases, and ratios of attraction basins of nondefection for the cases with more than two strategies. The study of the population has revealed that involvement of both punishments is more effective than one of the punishments alone in sustaining cooperation, based on two respects: larger scopes of the attraction basins of full nondefection and larger region sizes of nondefection phases. The mechanism behind this result can be attributed to the fact that abundantly available punishments against defectors are provided by the two types of punishers at different evolutionary stages, respectively. The analytical results have suggested a monotonic effect of the synergy effect, punishment fine, and feedback sensitivity on facilitating the advantages of nondefectors in terms of scopes of the attraction basins of full nondefection. Further support of the same effect is obtained by giving a comprehensive picture of the strategy fractions in the parameter plane (A, α) for six different evolution situations: When both A and α are large enough, nondefection phases appear since both frequently available punishments and the large cost of one punishment are imposed on defectors. Of particular note is that cooperator-driven punishment is overall noncompetitive in the presence of defector-driven punishment, however, slightly more favorable for nondefectors. In addition, punishment fines and feedback sensitivity turn out to be the key parameters to govern the performances of punishment measures in such a population. By means of the semianalytical method, we have given boundaries to accurately distinguish nondefection phases from full-defection phases for each evolutionary situation, through roughly estimating the fractions of different nondefective strategies near the boundaries. Nevertheless, traditional cooperators can undermine the evolutionary advantages of punishers [10–12], even though in the desirable situations that the cooperation is further promoted by both punishment measures together.

For a networked population, agent-based simulations of the evolutionary dynamics generate more rich results. In comparison to the findings under the infinite well-mixed condition, networked structure are overall more favorable for the survival or even dominance of nondefectors, i.e., sustaining the public cooperation, which is supported by the comparison of comprehensive pictures for two different populations. We have obtained the physical original of this phenomenon through a detailed statistical analysis of the emerging spatial patterns in terms of the frequencies of different types of strategies and edges, mean payoff gaps between the two ends of edges connecting two individuals with different strategies, net increase of different nondefectors, and payoff-gap and number spectra of different states of edges. The analysis has revealed that it can be attributed to two major factors: (1) support from network reciprocity in any case and (2) mutualism between traditional cooperators and punishers that works in the cases with traditional cooperation. Quite remarkably, the punishers help traditional cooperators to reduce the competitive ability of defectors in the vicinity of punishers' clusters to some extent, whereas traditional cooperators, who are effectively second-order free riding on the punishments, form an active layer around punishers, which protects them against defectors. Mutualism between the punishers and traditional cooperators is thus established. In particular, as a consequence of the strong mutualism, traditional cooperators can counterintuitively largely facilitate the prevalence of defector-driven punishers, while it turns out that cooperator-driven punishers are always vulnerable to the same second-order free riding. Another interesting point is that cooperator-driven punishment is proved to be a more powerful measure than defector-driven punishment with respect to enlarging the scopes of favorable parameters and promoting cooperation by virtue of their great ability to fully take advantage of network reciprocity. This finding is in accordance with the empirical conclusion that self-regulation is superior to government regulation for benefiting consumers, businesses, and the economy [32,37,78].

Moreover, unlike what happens in the infinite well-mixed population, to have a desirable evolutionary outcome with high-level cooperation in the network, an intermediate range of feedback sensitivity is surprisingly needed, while the monotonic effect of the synergy effect, punishment fine, and feedback sensitivity on facilitating the advantages of traditional cooperators rather than punishers is still identified. The statistical analysis of spatial pattern formations has also provided a physical understanding. For low sensitivity, punishments from unresponsive punishers are lacking, and thus unable to sustain nondefectors' survival. Conversely, if the sensitivity is high, overly sensitive punishers would punish too many defectors so that they cannot have competitive payoffs in comparison to defectors. Therefore, intermediate sensitivity is an optimal choice of punishers. We have found that under such a parameter condition punishers not only can defeat defectors through sufficient punishments but can also maintain competitive advantages to get clustering in time, leading to persistent growth of nondefectors' clusters. Furthermore, in the vicinity of the borders of nondefectors' clusters, isolated cooperator-driven punishers or those at the tip of peninsulas are found to be pioneers of expansions. In addition, it is also worth noting that our semianalytical approach fails to give a relatively accurate boundary in the situation with defectors and defector-driven punishers, given that feedback sensitivity is high. This discrepancy is attributed to the vicious circle arising from the strategic nature of defector-driven punishers, which also causes the poorer performance of defector-driven punishment in a networked population. Finally, explorations of the frequencies of different strategies as a function of feedback sensitivity or both the sensitivity and punishment fine have disclosed a competitive relationship among nondefectors, especially among cooperator-driven and defector-driven punishers. This is the result of the majoritylike rule which frequently happens within nondefectors' clusters.

Relating to reality, we conclude our work by providing two general remarks. First, our study has revealed potentially favorable conditions for operations of self-regulation and government regulation. More precisely, in social or economic systems with imperfect information induced by the spatial structure, self-regulation can be accepted as a useful tool to sustain commons and eliminate conflicts of interest, particularly when self-regulation organizations have an intermediate response speed and regulations that are strong enough. Conversely, if the state of the whole system is known to the individuals (like the Internet system), a mix of the two types of regulations may be a better choice to achieve optimal public goals, such as an Internet coregulation scheme [74]. From another perspective, our study has given a possible interpretation of why self-regulation has recently begun to gain wide acceptance and interest for applications [37,78–81]: imperfect information or spatial limitation in the markets or social systems. Second, we can conclude from our study that regulating effects from response speed of regulation organizations are highly dependent upon the information transparency (i.e., structure) of the systems. More precisely, high information transparency means that a high response speed is always essential for a good running market. In contrast, when information transparency is rather low because of spatial limitation, selecting an intermediate response speed is a better choice for regulation organizations such as SROs.

Finally, we must stress that our present model does not captures top-down prescriptive rules of government regulation, as well as third party certification schemes or government watchdogs [74], which may result in better firm behavior. For self-regulation, we avoid the adversarial problem of putting the fox in charge of the hen house, which is beyond our present research. Nevertheless, our present study has developed a computational and theoretical paradigm to understand the relative roles played by SROs and government regulation (an external powerful force such as troops) in the framework of game theory, which has potential implications not only in self-regulation but also in other topics in economics and political science. We hope to be able to extend our analysis to temporal networks [82] or multilayer networks [83–86], as well as to more complex situations by considering the above-mentioned realistic mechanisms or antisocial punishment [9,71–73].

ACKNOWLEDGMENTS

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APPENDIX A

The evolutionary dynamics of the studied system can be analytically described by a set of replicator equations [25,87]

$$\frac{df_s(t)}{dt} = f_s(t)(\Pi_s - \overline{\Pi}), \tag{A1}$$

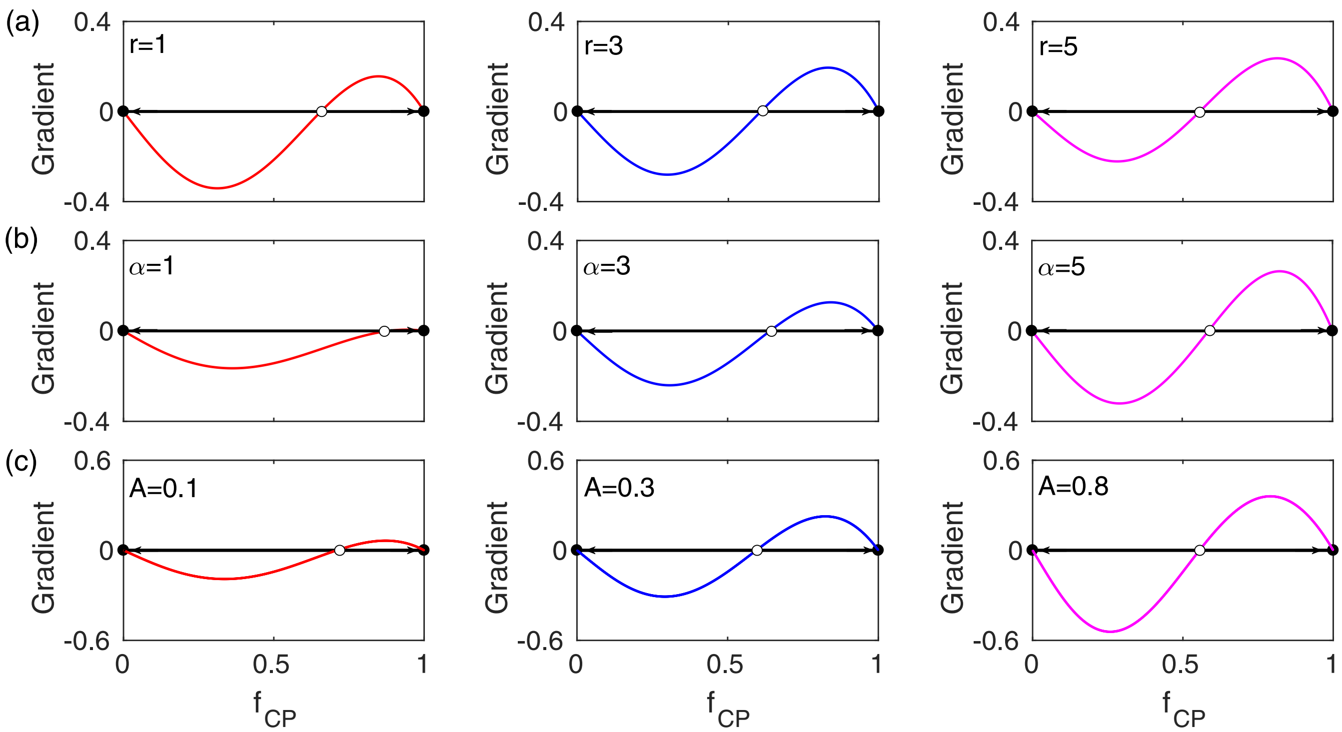

FIG. 14. Gradient of selection dependent on the fraction of cooperator-driven punishers for the evolutionary situation D + CP. Stable steady states $f_{CP} = 0$ and $f_{CP} = 1$ are depicted with closed circles, while the unstable steady state is depicted with an open circle. Arrows indicate the expected direction of evolution. The arrow pointing to the right indicates that cooperator-driven punishment is favored over defection. Results are shown for (a) three different values of r with $A = 0.25$ and $\alpha = 4.0$, (b) three different values of α with $A = 0.25$ and $r = 3.0$, and (c) three different values of A with $\alpha = 4.0$ and $r = 3.0$. In any case, the intermediate state is unstable when the left side of the gradient is negative while the right is positive.

where f_s and Π_s indicate the fractions of individuals possessing the strategy $s \in \{\mathrm{TC, D, CP, DP}\}$ and their correspondingly expected payoff in theoretical analysis, respectively, and $\overline{\Pi} = \sum_s f_s(t)\Pi_s$ is the average payoff of the entire population. Theoretically, the expected payoff for each strategy s could be further given by

$$\Pi_s = \sum_{0\leqslant N_i\leqslant G-1} \frac{(G-1)!}{\prod_i N_i!} \prod_i f_i^{N_i} \Pi_i'. \tag{A2}$$

Furthermore, taking the D + CP case as an example, Π_i' can be obtained according to the following method:

$$\Pi_{\mathrm{D}}' = (1.0 - g_{\mathrm{CP}})^{N_{\mathrm{CP}}}\left(\frac{rN_{\mathrm{CP}}}{G}\right) + [1.0 - (1.0 - g_{\mathrm{CP}})^{N_{\mathrm{CP}}}]\left(\frac{rN_{\mathrm{CP}}}{G} - \alpha\right), \quad g_{\mathrm{CP}} = \frac{AN_{\mathrm{CP}}}{G}; \tag{A3}$$

$$\Pi_{\mathrm{CP}}' = (1.0 - g_{\mathrm{CP}})\left(\frac{r(N_{\mathrm{CP}}+1)}{G} - 1.0\right) + g_{\mathrm{CP}}\left(\frac{r(N_{\mathrm{CP}}+1)}{G} - 1.0 - \sum_{i=0}^{N_{\mathrm{CP}}} \frac{N_{\mathrm{CP}}!}{i!(N_{\mathrm{CP}}-i)!} g_{\mathrm{CP}}^i (1.0 - g_{\mathrm{CP}})^{(N_{\mathrm{CP}}-i)} \frac{N_{\mathrm{D}}\alpha}{i+1}\right),$$

$$g_{\mathrm{CP}} = \frac{A(N_{\mathrm{CP}}+1)}{G}. \tag{A4}$$

We can easily extend the above method to the other five evolutionary situations.

APPENDIX B

The behaviors of the selection gradient df_{CP}/dt for different parameter conditions are presented in Fig. 14. The illustrations show that CP can transform the defined game into a coordination game with full CP and full D as the two stable equilibria, along with an intermediate unstable steady state, i.e., coexistence state of the two strategies. By means of the rule that a larger gradient indicates higher speeds at which the system converges to the stable equilibrator, we can state that FD is more attractive than full CP, regardless of large r (β and A) being able to help full CP be advantageous to some extent.

Likewise, as shown in Fig. 15, DP can still transform the defined game into a coordination game with full DP and

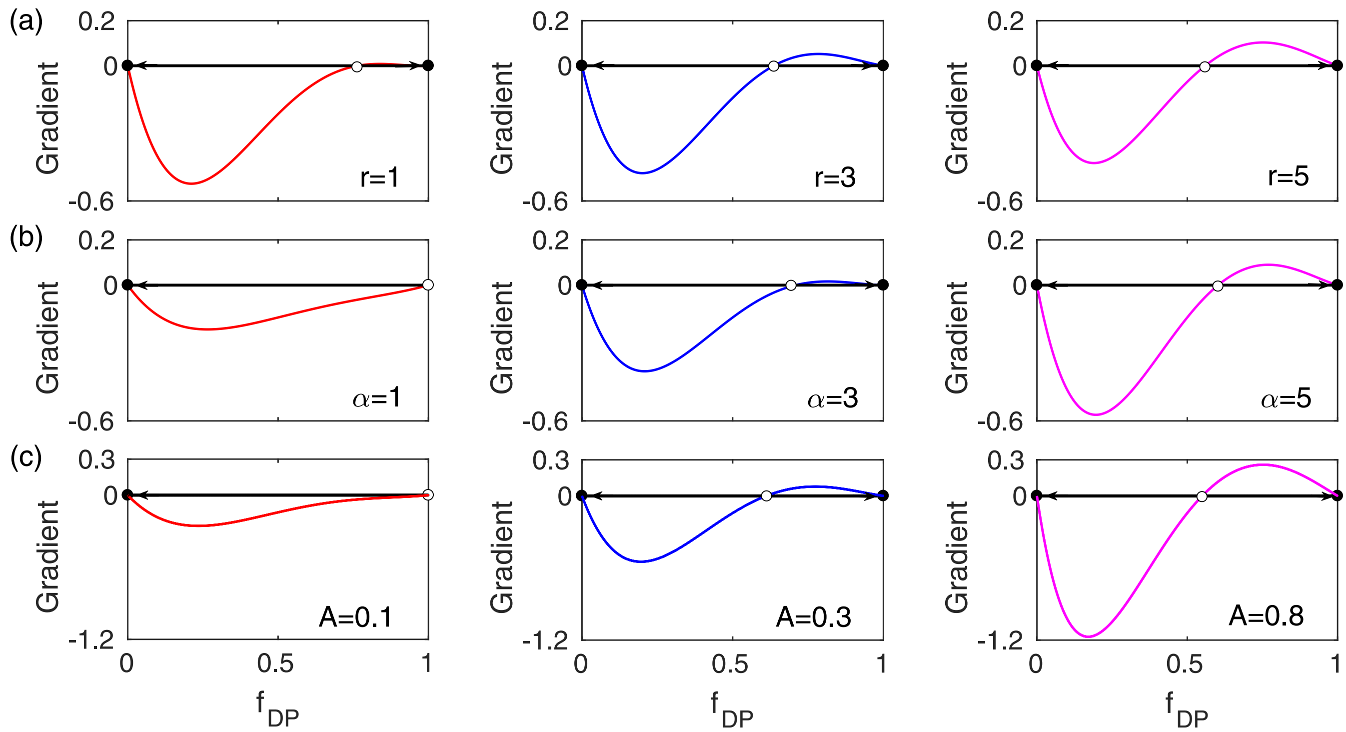

FIG. 15. Gradient of selection dependent on the fraction of defector-driven punishers for the evolutionary situation D + DP. In most cases, $f_{DP} = 0$ and $f_{DP} = 1$ are stable steady states, while the coexistence state of the two strategies is unstable or even impossible. Arrows indicate the expected direction of evolution; DP is favored over D if the arrow points to the right. Results are shown for (a) three different values of r with $A = 0.25$ and $\alpha = 4.0$, (b) three different values of α with $A = 0.25$ and $\alpha = 3.0$, and (c) three different values of A with $\alpha = 4.0$ and $r = 3.0$.

FD as the two stable equilibria, except for the left panel in Figs. 15(b) and 15(c). We also find that the system can more quickly reach the state of FD than that of full DP. Figures 14 and 15 provide a key hint that the position of the coexistence state can be used to measure how facilitative different parameters are for the punishment. More specifically, the lower the intermediate coexistence state's position value is, the more likely the system is to reach the state of full punishment.

Figure 16 provides a comprehensive picture of the system dynamics for the TC + D + DP case, exhibiting rich phenomena. Depending on the initial conditions, the system will evolve towards one of the following three states, FD, stable coexistence of DP and TC (i.e., state of segment punishers), and full DP, except for the state of full cooperation (FC) or coexistence of the three strategies. It is obvious that the three strategies fail to form a cyclic dominance [73]. Note that the attraction basin of nondefection gets larger with increasing r, α, or A, which further confirms that the monotonic effects of the three parameters in promoting public goods is in spite of the intervention of traditional cooperation. Specifically, achieving a segment punishment (SP) state largely depends on whether there are adequate initial defector-driven punishers or not, especially for large r, α, or A. Another obvious feature is that defectors only have the opportunity to completely conquer an entire population after defector-driven punishers have gone extinction (i.e., punishment is absent), resulting from exploitation of traditional cooperators, exhibiting that the evolution trajectories which ended in the FD state have to first reach the side of simplex with two corners D and TC.

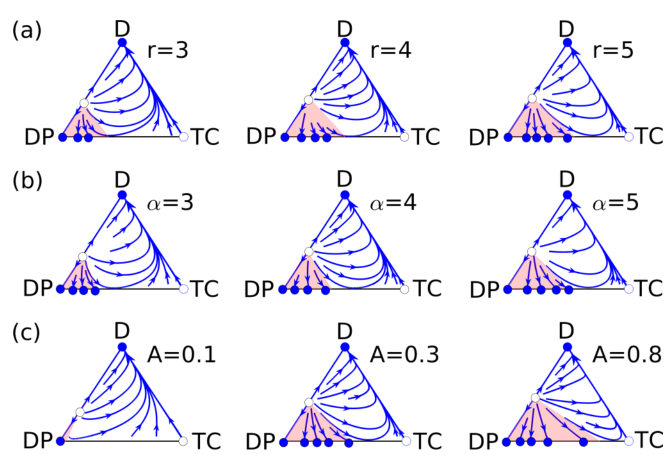

FIG. 16. Phase portraits of the system for the evolutionary situation TC + D + DP for (a) three representative values of the synergy factor, with the parameters $A = 0.25$ and $\alpha = 4.5$; (b) three representative values of the punishment fine, with the parameters $A = 0.25$ and $r = 4.5$; and (c) three representative values of sensitivity, with the parameters $\alpha = 4.5$ and $r = 4.5$. The vertices marked by a closed circle indicate attractors of the system, while those marked by open circles indicate repellers. The light red regions indicate attraction basins where the system converges to a state of either full DP or SP.

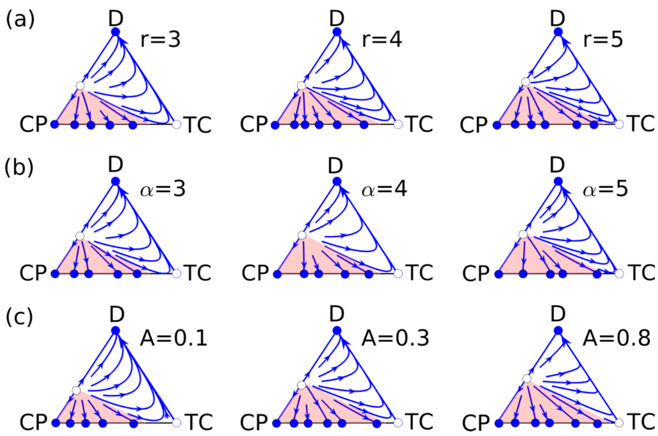

FIG. 17. Phase portraits of the system for the evolutionary situation TC + D + CP for (a) three representative values of the synergy factor, with the parameters $A = 0.25$ and $\alpha = 4.5$; (b) three representative values of the punishment fine, with the parameters $A = 0.25$ and $r = 4.5$; and (c) three representative values of sensitivity, with the parameters $\alpha = 4.5$ and $r = 4.5$. The vertices marked by a closed circle indicate attractors of the system, while those marked by open circles indicate repellers. The light red regions indicate attraction basins where the system converges to a state of either full CP or SP.

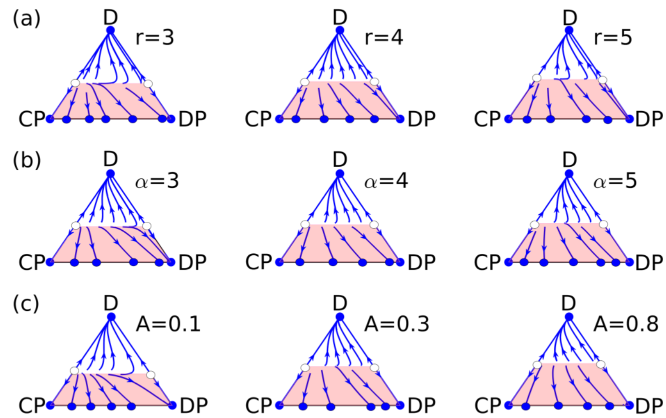

FIG. 18. Phase portraits of the system for the evolutionary situation TC + CP + DP for (a) three representative values of the synergy factor, with the other parameters $A = 0.25$ and $\alpha = 4.5$; (b) three representative values of the punishment fine, with the other parameters $A = 0.25$ and $r = 4.5$; and (c) three representative values of sensitivity, with the parameters $\alpha = 4.5$ and $r = 4.5$. The three vertices marked by closed circles suggest that each of the three strategies is an attractor of the system. The light red regions indicate attraction basins where the system converges to a state of FP (full CP or full DP) or SP.

As shown in Fig. 17, three strategies (TC, D, and CP) together generate similar results especially with respect to both the patterns of attraction basins and evolution trajectories. The only difference is that CP performs better in facilitating more favorable initial conditions under which the system evolves towards the SP state. Thus, combined with the results given by Fig. 16, CP shows a slightly greater advantage than DP in sanctioning those defectors. Nevertheless, Figs. 16 and 17 show that the basin of attraction for SP is less than half of the simplex, which reveals that one of the two punishers alone fails to sustain cooperation unless the punishers initially capture the majority of the population since both punishments are challenged by second-order free riding from traditional cooperators.

Figure 18 provides a different picture in which cooperator-driven and defector-driven punishers together can effectively repel defectors in most cases. More surprisingly, this phenomenon is robust to the changes of punishment fine, synergy factor, and feedback sensitivity, and increasing the three parameters can enhance competitive advantage of the two punishers to some extent. This suggests a nontrivial interplay between defector-driven and cooperator-driven punishment in promoting public cooperation (including TC, CP, and DP). Still, a stable interior point is absent in such a case. Based on the illustrations in Fig. 18, we can say that a mix of the two types of punishment is better at preventing the entire population from being eroded by the first-order free riders.

In a similar way, the growth of DP clusters starts with individuals at the tips of peninsulas and nearby isolated ones, as shown in Fig. 19. In the same way, low or high sensitivity induces noncompetitive defector-driven punishers. Intermediate sensitivity is optimal for the punishers.

Figures 20 and 21 further support that there exists an optimal intermediate range of feedback sensitivity to facilitate the complete dominance of defector-driven punishers in the presence of traditional cooperators, which is also considered as a desirable evolutionary outcome with both first-order and second-order free riding. As shown in Fig. 20, traditional cooperators at the borders of clusters still play the role of a protective film, which spatially isolates the punishers bearing the punishment cost from those defectors. Meanwhile, this gives these traditional cooperators a chance to beat their defective neighbors whose payoffs have been greatly reduced by DP. Compared with the TC + D + CP case, the spatial pattern formations shown in Figs. 20 indicate a similar trajectory of evolution. However, except in the case of large intermediate feedback sensitivity, punishers are too active to lose the territorial battle with defectors, further leading to the extinction of all nondefectors. In the same way, there is a strong mutualism between defector-driven punishers and traditional cooperators.

The results for the evolutionary situation D + CP + DP are presented in Fig. 22, enabling a direct comparison between the two types of punishers. The illustrations are consistent with the results in Fig. 6(e) that cooperator-driven punishers are prior to defector-driven punishers. In particular, positive peaks of both $\Delta_{\text{CP-D}}(t)$ and $\Delta n_{\text{CP-D}}(t)$ are found to be larger than those of both $\Delta_{\text{DP-D}}(t)$ and $\Delta n_{\text{DP-D}}(t)$ for the same parameter settings, while those of $\Delta n_{\text{DP-CP}}(t)$ are always approximately zero. The phenomena reveal that the cooperator-driven punishers' prevalence depends mainly on them being more successful in the battle against defectors. Thus $\Delta_{\text{CP-D}}(t)$ [$\Delta n_{\text{CP-D}}(t)$] is larger than $\Delta_{\text{DP-D}}(t)$ [$\Delta n_{\text{DP-D}}(t)$] (see the third and fourth rows in Fig. 22). However, in the areas without defectors their competition still frequently happens in the form of majoritylike rule, leading to fluctuations of $\Delta n_{\text{DP-CP}}(t)$ exhibited in Fig. 22 and further exacerbating the divide between the two types of punishers. At the same

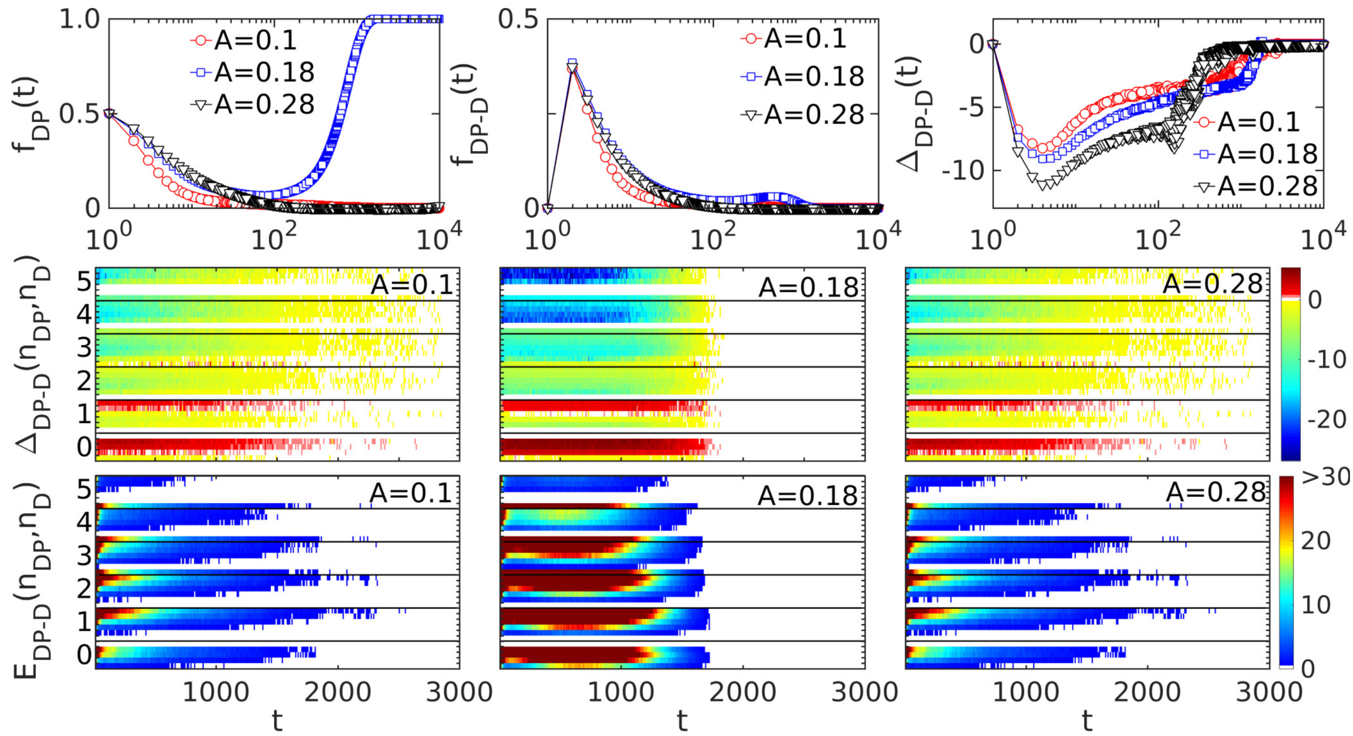

FIG. 19. Illustrations of the evolutionary dynamics of the networked populations in the D + DP case for three representative values of A. The behaviors of the five different statistical characteristic quantities are shown. The other parameters are $r = 4.0$ and $\alpha = 4.0$.

time, cooperator-driven punishers seem more essential for the survival of defector-driven punishers since DPs cannot persist alone and they have to combine with CPs who can fully take advantage of network reciprocity.

Using the same initial conditions for Fig. 4, in Fig. 23 we present the comprehensive picture in the parameter plane (A, α) of the evolutionary dynamics, as well as semianalytically estimated boundary lines. Larger regions of ND phases

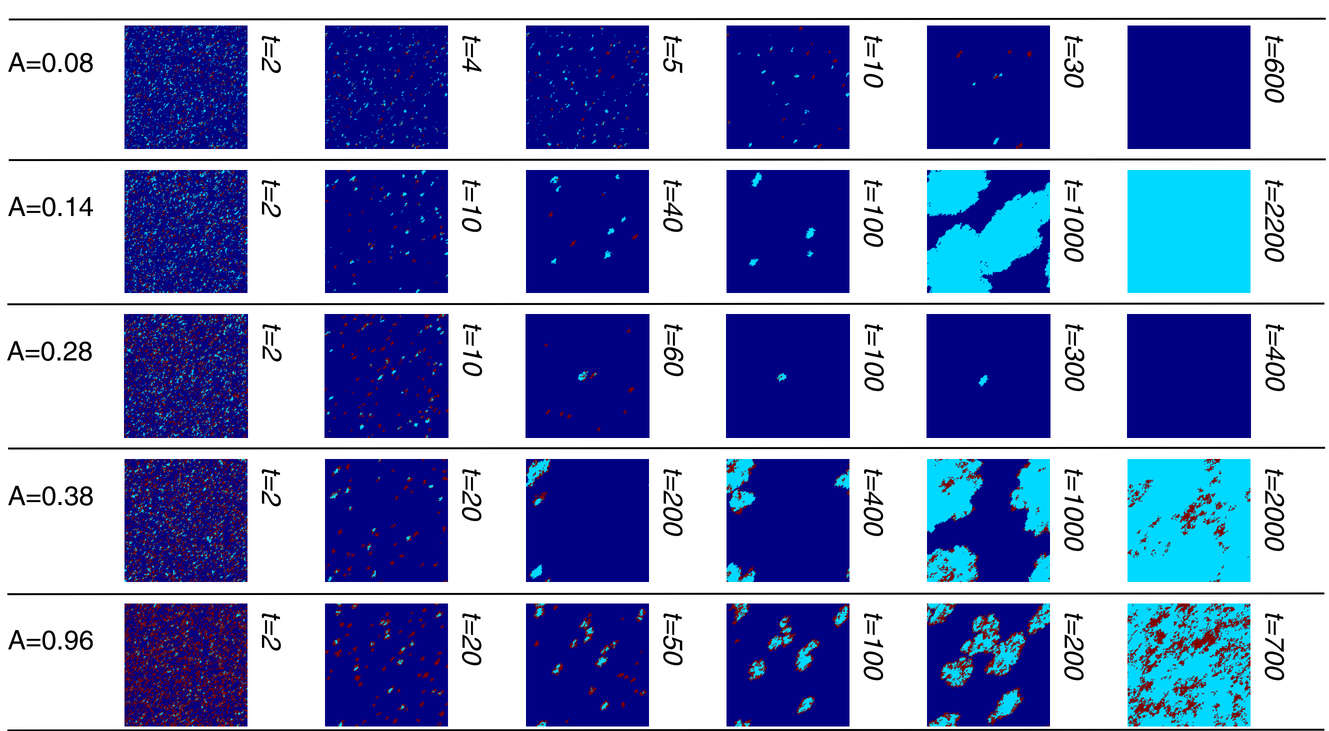

FIG. 20. Spatial evolution of the three competing strategies TC, D, and DP, in networked populations, for four representative values of A. Snapshots of the hexagonal lattice with size $L = 200$ are depicted for a punishment fine of $\alpha = 4.4$. The color codes are the same as in Fig. 10.

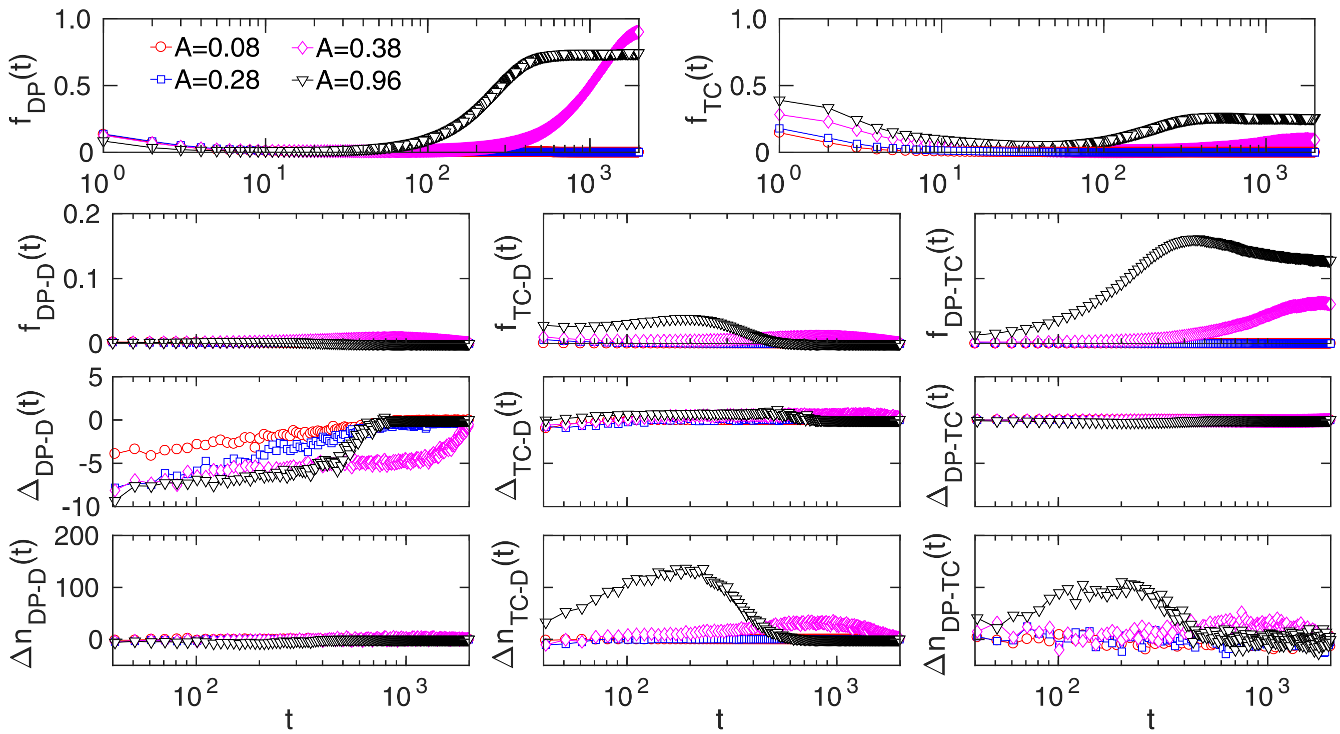

FIG. 21. Roles of different strategies in the evolutionary process in networked populations for three competing strategies TC, D, and DP. The behaviors of three classes of statistical characteristic quantities are shown for four representative values of A corresponding to the four groups of spatial snapshots presented in Fig. 20. The other parameters are $r = 4.0$ and $\alpha = 4.4$.

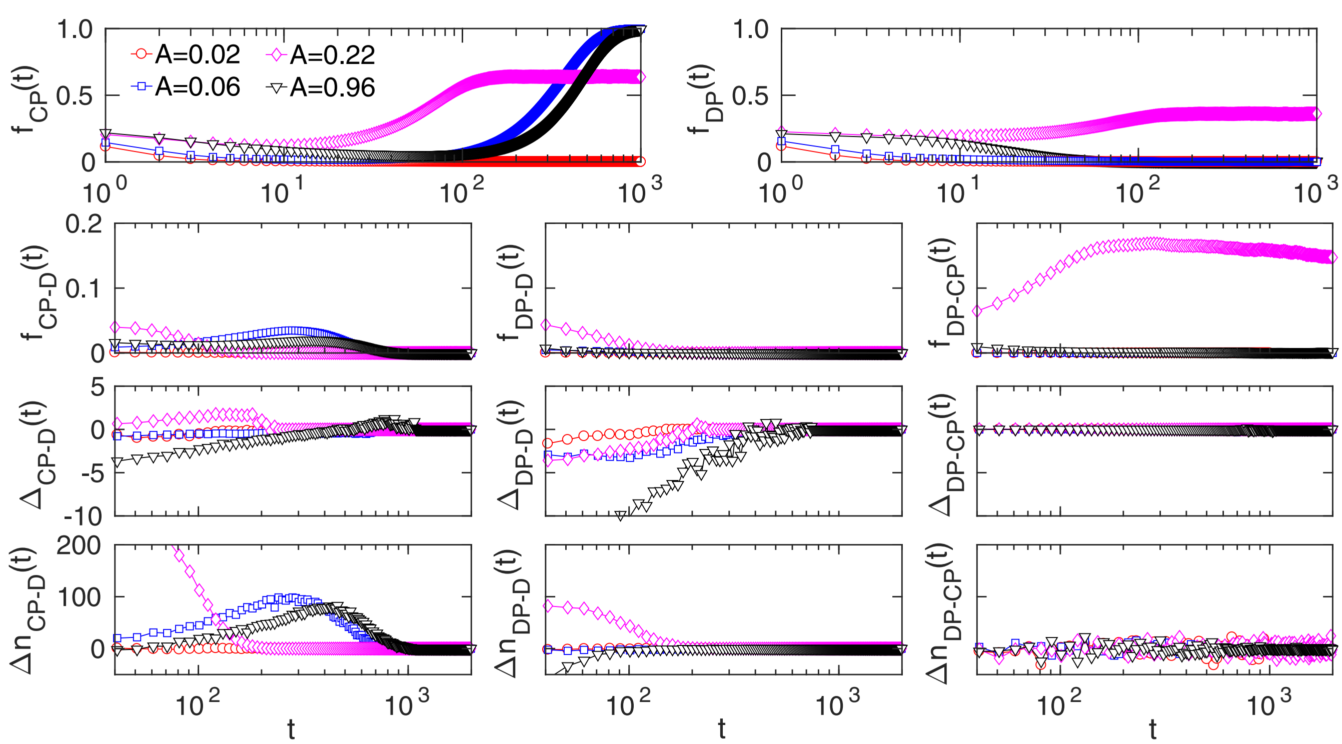

FIG. 22. Roles of different strategies in the evolutionary process in networked populations for the three strategies D, CP, and DP. The behaviors of three classes of statistical characteristic quantities are shown for four representative values of A. The other parameters are $r = 4.0$ and $\alpha = 4.0$.

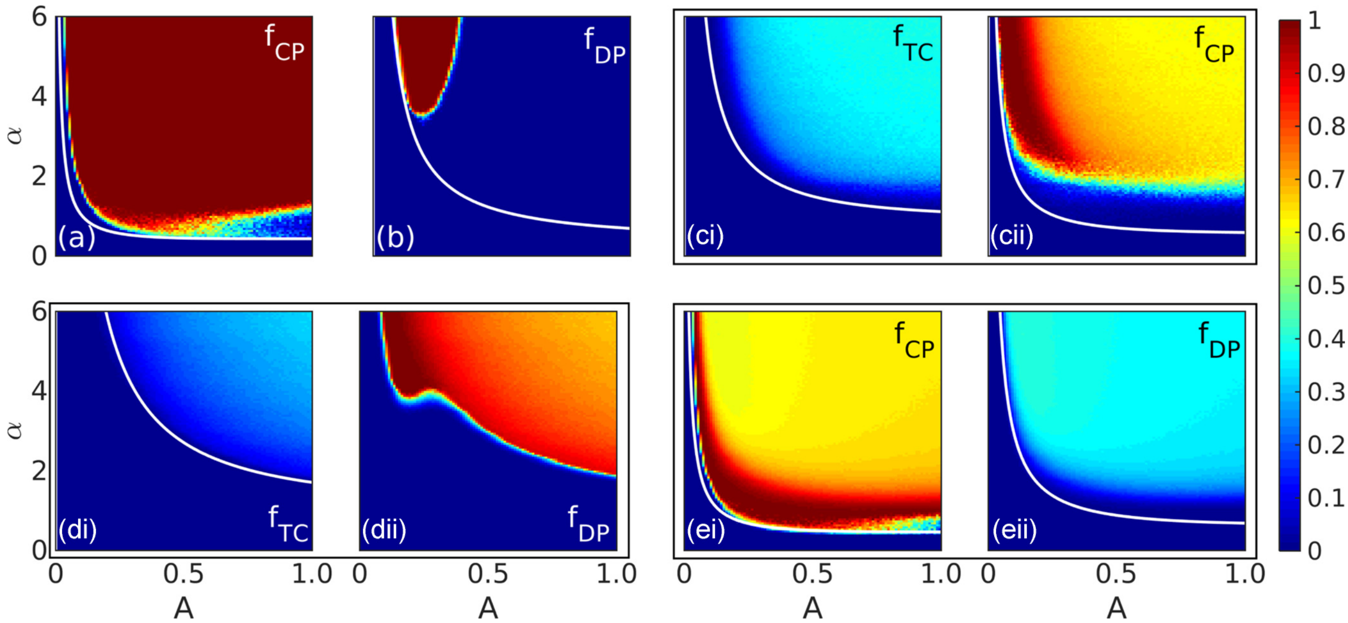

FIG. 23. Dependence of simulated final steady fractions of three nondefection strategies on both A and α; the results obtained from networked populations are illustrated for five different evolution situations: (a) D + CP, (b) D + DP, (c) TC + D + CP, (d) TC + D + DP, and (e) D + CP + DP. In all cases, the value of the synergy factor is $r = 4.0$. Correspondingly, the value of f_s used to semianalytically estimate the boundary lines are (see Appendix C for further details) (a) $f_{\rm CP} = 1.0$, (b) $f_{\rm DP} = 0.96$, (c i) $f_{\rm C} = 0.1$ and $f_{\rm CP} = 0.55$, (c ii) $f_{\rm C} = 0.1$ and $f_{\rm CP} = 0.71$, (d i) $f_{\rm TC} = 0.115$ and $f_{\rm DP} = 0.465$, (e i) $f_{\rm CP} = 0.1$ and $f_{\rm DP} = 0.84$, (e ii) $f_{\rm CP} = 0.3$ and $f_{\rm DP} = 0.45$.

suggest that network reciprocity induced by network structure largely benefits cooperation by allowing nondefectors to organize themselves into compact clusters.

APPENDIX C

This Appendix presents how to semianalytically obtain the boundaries separating SP and the FD phase at which the relationship

$$\Pi_x = \Pi_{\rm D} \tag{C1}$$

is satisfied, while Π_x can be obtained as

$$\Pi_x = \sum_i w_s \Pi_s, \tag{C2}$$

where w_s represents the contribution weight of population with strategy $s \in \{\text{TC, D, CP, DP}\}$ in resisting defection. Furthermore, we can estimate the values of f_s and w_s based on either the initial values of f_s for numerical integration of the equations given in Appendix A for analytical treatments or the proportions of different strategy populations near the boundaries between the two phases for simulations so as to semianalytically identify the boundary lines for corresponding evolution situations. In more detail, combining Eqs. (C1) and (C2) with the theoretical expressions of payoffs for different strategies in Appendix A, we further accordingly give the expressions of α at boundary lines as a function of A for the following six evolution situations. (1) For D + CP,

$$\alpha = \frac{\frac{r}{G} - 1}{E\psi + E(1.0 - g'_{\rm CP})^i - 1}, \tag{C3}$$

where

$$E = \sum_{i=0}^{G-1} \frac{(G-1)!}{i!(G-1-i)!} f_{\rm CP}^i (1.0 - f_{\rm CP})^{G-1-i}, \quad \psi = \sum_{j=0}^{i} \frac{i!}{j!(i-j)!} g_{\rm CP}^{j+1} (1.0 - g_{\rm CP})^{i-j} \frac{G-1-i}{j+1},$$

$$g'_{\rm CP} = \frac{Ai}{G}, \quad g_{\rm CP} = \frac{A(i+1)}{G}.$$

It should be noted that E and ψ are just two operators to make the equation look short, rather than functions or something else. Moreover, f_s indicates the proportion of population of strategy s at the boundary lines. (2) For D + DP,

$$\alpha = \frac{\frac{r}{G} - 1}{E\psi + E(1.0 - g'_{\rm DP})^i - 1}, \tag{C4}$$

where

$$E = \sum_{i=0}^{G-1} \frac{(G-1)!}{i!(G-1-i)!} f_{\mathrm{DP}}^{i} (1.0 - f_{\mathrm{DP}})^{G-1-i}, \quad \psi = \sum_{j=0}^{i} \frac{i!}{j!(i-j)!} g_{\mathrm{DP}}^{j+1} (1.0 - g_{\mathrm{DP}})^{i-j} \frac{G-1-i}{j+1},$$

$$g'_{\mathrm{DP}} = \frac{A(G-i)}{G}, \quad g_{\mathrm{DP}} = \frac{A(G-i-1)}{G}.$$

(3) For TC + D + CP,

$$\alpha = \frac{\frac{r}{G} - 1}{w_{\mathrm{CP}} E \psi + E(1.0 - g'_{\mathrm{CP}})^{i} - 1}, \tag{C5}$$

where

$$w_{\mathrm{CP}} = \frac{f_{\mathrm{CP}}}{f_{\mathrm{TC}} + f_{\mathrm{CP}}}, \quad E = \sum_{i,j=0}^{G-1} \frac{(G-1)!}{i!j!(G-1-i-j!)!} f_{\mathrm{CP}}^{i} f_{\mathrm{TC}}^{j} (1.0 - f_{\mathrm{CP}} - f_{\mathrm{TC}})^{G-1-i-j},$$

$$\psi = \sum_{k=0}^{i} \frac{i!}{k!(i-k)!} g_{\mathrm{CP}}^{k+1} (1.0 - g_{\mathrm{CP}})^{i-k} \frac{G-1-i-j}{k+1}, \quad g'_{\mathrm{CP}} = \frac{A(i+j)}{G}, \quad g_{\mathrm{CP}} = \frac{A(i+j+1)}{G}.$$

(4) For TC + D + DP,

$$\alpha = \frac{\frac{r}{G} - 1}{w_{\mathrm{DP}} E \psi + E(1.0 - g'_{\mathrm{DP}})^{i} - 1}, \tag{C6}$$

where

$$w_{\mathrm{DP}} = \frac{f_{\mathrm{DP}}}{f_{\mathrm{TC}} + f_{\mathrm{DP}}}, \quad E = \sum_{i,j=0}^{G-1} \frac{(G-1)!}{i!j!(G-1-i-j!)!} f_{\mathrm{DP}}^{i} f_{\mathrm{TC}}^{j} (1.0 - f_{\mathrm{DP}} - f_{\mathrm{TC}})^{G-1-i-j},$$

$$\psi = \sum_{k=0}^{i} \frac{i!}{k!(i-k)!} g_{\mathrm{DP}}^{k+1} (1.0 - g_{\mathrm{DP}})^{i-k} \frac{G-1-i-j}{k+1},$$

$$g'_{\mathrm{DP}} = \frac{A(G-i-j)}{G}, \quad g_{\mathrm{DP}} = \frac{A(G-i-j-1)}{G}.$$

(5) For D + CP + DP,

$$\alpha = \frac{\frac{r}{G} - 1}{w_{\mathrm{DP}} E \psi_{\mathrm{DP}} + w_{\mathrm{CP}} E \psi_{\mathrm{CP}} + E(1.0 - g'_{\mathrm{CP}})^{i} (1.0 - g'_{\mathrm{DP}})^{j} - 1}, \tag{C7}$$

where

$$w_{\mathrm{DP}} = \frac{f_{\mathrm{DP}}}{f_{\mathrm{CP}} + f_{\mathrm{DP}}}, \quad w_{\mathrm{CP}} = \frac{f_{\mathrm{CP}}}{f_{\mathrm{CP}} + f_{\mathrm{DP}}},$$

$$E = \sum_{i,j=0}^{G-1} \frac{(G-1)!}{i!j!(G-1-i-j!)!} f_{\mathrm{CP}}^{i} f_{\mathrm{DP}}^{j} (1.0 - f_{\mathrm{CP}} - f_{\mathrm{DP}})^{G-1-i-j},$$

$$\psi_{\mathrm{DP}} = \sum_{k=0}^{i} \frac{i!}{k!(i-k)!} g_{\mathrm{CP}}^{k} (1.0 - g_{\mathrm{CP}})^{i-k} \sum_{l=0}^{j} \frac{j!}{l!(j-l)!} g_{\mathrm{DP}}^{l+1} (1.0 - g_{\mathrm{DP}})^{j-l} \frac{G-1-i-j}{k+l+1},$$

$$\psi_{\mathrm{CP}} = \sum_{k=0}^{i} \frac{i!}{k!(i-k)!} g_{\mathrm{CP}}^{k+1} (1.0 - g_{\mathrm{CP}})^{i-k} \sum_{l=0}^{j} \frac{j!}{l!(j-l)!} g_{\mathrm{DP}}^{l} (1.0 - g_{\mathrm{DP}})^{j-l} \frac{G-1-i-j}{k+l+1},$$

$$g'_{\mathrm{DP}} = \frac{A(G-i-j)}{G}, \quad g'_{\mathrm{CP}} = \frac{A(i+j)}{G}, \quad g_{\mathrm{DP}} = \frac{A(G-i-j-1)}{G}, \quad g_{\mathrm{CP}} = \frac{A(i+j+1)}{G}.$$

(6) For TC + D + CP + DP,

$$\alpha = \frac{\frac{r}{G} - 1}{w_{\mathrm{CP}} E \psi_{\mathrm{CP}} + w_{\mathrm{DP}} E \psi_{\mathrm{DP}} + E(1.0 - g'_{\mathrm{CP}})^{j} (1.0 - g'_{\mathrm{DP}})^{k} - 1}, \tag{C8}$$

where

$$w_{\mathrm{DP}} = \frac{f_{\mathrm{DP}}}{f_{\mathrm{TC}} + f_{\mathrm{CP}} + f_{\mathrm{DP}}}, \quad w_{\mathrm{CP}} = \frac{f_{\mathrm{CP}}}{f_{\mathrm{TC}} + f_{\mathrm{CP}} + f_{\mathrm{DP}}},$$

$$E = \sum_{i,j=0}^{G-1} \frac{(G-1)!}{i!j!k!(G-1-i-j-k)!} f_{\mathrm{TC}}^i f_{\mathrm{CP}}^j f_{\mathrm{DP}}^k (1.0 - f_{\mathrm{TC}} - f_{\mathrm{CP}} - f_{\mathrm{DP}})^{G-1-i-j-k},$$

$$\psi_{\mathrm{CP}} = \sum_{l=0}^{j} \frac{j!}{l!(j-l)!} g_{\mathrm{CP}}^{l+1} (1.0 - g_{\mathrm{CP}})^{j-l} \sum_{m=0}^{k} \frac{k!}{m!(k-m)!} g_{\mathrm{DP}}^{m} (1.0 - g_{\mathrm{DP}})^{k-m} \frac{G-1-i-j-k}{l+m+1},$$

$$\psi_{\mathrm{DP}} = \sum_{l=0}^{j} \frac{j!}{l!(j-l)!} g_{\mathrm{CP}}^{l} (1.0 - g_{\mathrm{CP}})^{j-l} \sum_{m=0}^{k} \frac{k!}{m!(k-m)!} g_{\mathrm{DP}}^{m+1} (1.0 - g_{\mathrm{DP}})^{k-m} \frac{G-1-i-j-k}{l+m+1},$$

$$g'_{\mathrm{DP}} = \frac{A(G-i-j-k)}{G}, \quad g'_{\mathrm{CP}} = \frac{A(i+j+k)}{G}, \quad g_{\mathrm{DP}} = \frac{A(G-i-j-k-1)}{G}, \quad g_{\mathrm{CP}} = \frac{A(i+j+k+1)}{G}.$$

However, Figs. 4, 5(d), 5(e), 12, and 13 show that it is very hard to get accurate values of the proportions for different strategies, i.e., the values of w_s, because of high fluctuations near the boundaries between the SP and FD phases, especially for simulation cases. Therefore, the above equations actually provide a semianalytical method to identify the boundary lines, because one has to estimate the values of f_s so as to obtain a line which is as close as possible to the numerical boundaries.

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